Uncertainty in Crisis Bargaining with Multiple Policy Options*

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Abstract

Here, we examine the relationship between private capabilities and war in a new class of crisis bargaining games. Whereas traditional models consider only interactions that end in either war or a peaceful bargain, we assume that actors can also engage in low-level, costly policy options that shape final political outcomes in their favor (e.g., sanctions, airstrikes, cyberattacks). We analyze this new class of models using the tools of mechanism design. In contrast with the standard monotonicity results in crisis bargaining models, we identify general conditions under which a greater private willingness to fight is associated with a lower probability of war or expected utility from settlement. Theoretically, our results identify when including multiple conflict options breaks key tenets regarding the role of private information in conflict. In addition to this theoretical contribution, we also offer evidence that improving specific conflict capabilities can perversely lead to more war or worse outcomes for the state making the improvements.

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“Our traditional approach is either we’re at peace or at conflict. And I think that’s insufficient to deal with the actors that actually seek to advance their interests while avoiding our strengths.”

—General Dunford (2016), Chairman of the U.S. Joint Chiefs of Staff.

The relationship between private information and war is central to contemporary international relations theory (Fearon, 1995; Powell, 1999; Gartzke, 1999; Schultz, 1999; Wagner, 2000; Slantchev, 2003b; Meirowitz et al., 2008; Chassang and Padró i Miquel, 2009; Acharya et al., 2015; Ramsay, 2017; Gurantz and Hirsch, 2017; Spaniel and Bils, 2018; Spaniel and Malone, 2019). Theories of how private information affects war are typically developed using crisis bargaining games, in which states engage in a series of negotiations over a divisible asset and each state may go to war as an outside option. Some examples of these models include Brito and Intriligator (1985), Powell (1989), Morrow (1989), Fearon (1995), Wittman (2009), and Benson et al. (2016). These models assume that one or more states receive a private signal about their war payoffs, representing their hidden capabilities or willingness to go to war. This signal is commonly referenced as a state’s private type, as the state’s capabilities are known to them but unknown to their adversaries. Across the diverse set of models of crisis bargaining, two monotonicity results consistently emerge. First, when an actor privately knows that they would perform better should a war arise (i.e. an actor receives a better private signal about its war payoffs), negotiations are more likely to end in war. Second, when an actor receives a better private signal about its war payoffs, that actor will attain a greater final expected utility. In two foundational papers, Banks (1990) and Fey and Ramsay (2011) assess just how ubiquitous these monotonicity results are. Using the tools of mechanism design, they find that the monotonic relationships between private type and outcomes are not specific to any particular bargaining game form or equilibrium, but instead emerge in every equilibrium of any crisis bargaining game.

The crisis bargaining framework has shaped the theoretical study of the causes of war, but the traditional models it has produced rest on an implausible binary assumption: that actors either reach a peaceful and efficient bargain, or else engage in full-scale war (or some similar inefficient engagement). Recent research has relaxed this dichotomy, recognizing that crisis actors in practice face a vast array of policy options between a completely efficient peace and a decisive, destructive war. These policy options include implementing sanctions or tariffs (Coe, 2014; McCormack and Pascoe, 2017; Spaniel and Malone, 2019; Joseph, 2020); offering

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1We use the broad definition of crisis bargaining games from Fey and Ramsay (2011).
2The monotonicity results do not hold for models—for example, Slantchev (2011)—that consider an alternate form of uncertainty than Fey and Ramsay (2011).
third-party support to an adversary’s enemies (Kadera et al., 2003; Bapat, 2006; Schultz, 2010; Berman and Lake, 2019; Qiu, 2020; Chinchilla, 2021); pursuing a wide range of low-level operations sometimes classified as gray zone conflict (Mazarr, 2015; Gannon et al., 2020), hybrid conflict (Lanoszka, 2016) or hassling (Schram, 2021a,b); engaging in cyberwarfare (Gartzke and Lindsay, 2015; Baliga et al., 2020); entering into brinkmanship (Powell, 1989, 2015); and arming (Schultz, 2010; Debs and Monteiro, 2014; Gurantz and Hirsch, 2017; Coe and Vaynman, 2020). The new literature that considers multiple policy options highlights a troubling prospect for the foundations of international relations theory. This paper seeks to remedy this shortcoming. While the crisis bargaining literature has previously established very general theoretical conclusions about the relationship between private information and war, these results fall short when addressing settings where policymakers possess any coercive policy options outside of war and peace—like environments where states can go to war, stay at peace, or sanction their opponents.

Here, we re-examine the relationship between private information and war within what we call flexible-response crisis bargaining games. These games share the key features of crisis bargaining games, but they also allow for multiple forms of costly conflict short of all-out war. We find that previously established monotonicity results for crisis bargaining models, such as the relationship between greater private war payoffs and a greater final likelihood of war, break down in the flexible-response models. Using a game-free analysis along the lines of previous mechanism design research (Banks, 1990; Fey and Ramsay, 2011; Akçay et al., 2012; Spaniel, 2020; Liu, 2021), we show that these differences are not attributable to any particular game form or functional form but can arise in any flexible-response crisis bargaining game, depending on particular assumptions about modeling primitives and the strategies available to players short of war.

Using our flexible-response crisis bargaining games model, we identify new results on the relationship between private information and war. This is due in large part to our novel handling of private information. As we show here, once we allow for flexible policy options in between peace and war, the concept of a private willingness to go to war—a concept that is explored in depth in the standard crisis bargaining literature—becomes more complicated. In standard crisis bargaining models, private information is unidimensional, only affecting the payoffs from war. However, private war payoffs may be associated with greater or lower payoffs from alternate policy options. For example, the things that would make a state better at war might also affect its success with these intermediate policy options. For example, if a state has a wide range of privately known cyber-exploits, then the state knows that it could not only perform well in a conventional war that uses cyberattacks but also in a precise
cyberattack against a target’s infrastructure. In other words, because this state possesses strong private cyber-capabilities, both war and low-level conflict could be good options for them. In another scenario, if leadership is privately concerned about losing popular domestic support, then leadership may be more willing to fight a war to create a rally-around-the-flag effect than they would be to choose intermediate options such as implementing tariffs or carrying out a covert low-level attack (Baker and Oneal, 2001; Chapman and Reiter, 2004); in this scenario, a strong private desire to garner domestic support could make war a better option than low-level conflict or economic warfare. The question of whether private war payoffs are associated with greater or lower payoffs from alternative policy options is an empirical one, and its answer varies across cases and contexts, as we discuss below. What is important for our purposes is that these linkages undoubtedly exist, and their effects on the outcomes of crisis bargaining have not been systematically examined.

Flexible-response crisis bargaining games consider two competing players, a challenger and a defender, in a crisis. These games begins with the challenger either choosing not to transgress or selecting some level of transgression—an act that is beneficial to the challenger but hurts the defender (following Gurantz and Hirsch’s (2017) use of the term). In response, the defender can chose one of several options: they can enter into a decisive war over the transgression, can allow the transgression to come to fruition, or (following Schram’s (2021a) use of the term) can “hassle” and undercut the transgression through some destructive and non-decisive low-level response. How the defender plays the game is a function of their private type, which influences their war and hassling payoffs. Beyond this, we place no particular structure on the game. Within flexible-response crisis bargaining games, states could bargain, send costly or non-costly signals, make ultimatum offers, walk back or increase their selected hassling levels, or some combination of any of these actions before the game eventually ends.

As a proof of concept to illustrate how adding additional conflict options can undermine the canonical monotonicity results, consider the models in Figures 1 and 2. Both games are between a challenger (C) and defender (D). Nature moves first and decides whether D’s war payoff is low (θ = θ) or high (θ = θ̄), with each outcome having positive probability. D observes their private type, while C only knows the prior probability that D is a high or low type. Next, C selects whether to transgress (t = 1) or not (t = 0). Finally, D observes C’s choice, and makes their own choice in response: they can accept the transgression, go to war over the transgression, or conduct some limited response via hassling.

3 A transgression is isomorphic to an endogenous shift in the observable balance of power.
Figure 1 illustrates a setting where only the weak type of D goes to war, contrary to the standard result that the probability of conflict increases with the private value of war payoffs (Banks, 1990; Fey and Ramsay, 2011). This happens because the type with a greater war payoff is also more effective at hassling, as in the cyber-capability example above. More specifically, while war outcomes improve with $\theta$, hassling outcomes improve at an even faster rate. In equilibrium, the stronger type opts not to fight because hassling is even better. If the hassling option were not available, then the equilibrium would conform to the usual monotonicity findings, with both types going to war.

![Figure 1: Greater private type $\theta$ implies less war.](image)

C’s payoffs are listed first. Note here that $\bar{\theta}$ has both greater wartime payoffs and hassling payoffs relative to $\theta$. In equilibrium, C will transgress ($t = 1$), $\overline{\theta}$ D’s will go to war and $\bar{\theta}$ D’s will hassle.

Next, Figure 2 depicts a game in which the stronger type of D has a lower equilibrium payoff, in contrast to the standard result that an actor’s payoffs in the game are increasing in their private wartime payoffs (Banks, 1990; Fey and Ramsay, 2011). This happens because greater private willingness to fight corresponds to lower hassling ability, as in the rally-round-the-flag example above. In equilibrium here, $\bar{\theta}$ hassles and $\overline{\theta}$ fights. The improvement in the stronger type’s war payoff is not enough to offset the decrease in its hassling payoff. Once more, if the hassling option were unavailable, we would recover the standard pattern: both types would fight in equilibrium, with the stronger one yielding a greater utility.

These two models illustrate how introducing multiple related conflict options can undermine the monotonicity observed in standard crisis bargaining models. Of course, these simple models leave much to be desired. While it would be straightforward to create more generalized models than those discussed above, such an exercise would be inherently limited. When
we analyze any one game form, we cannot be sure which of our results are general features of crisis bargaining and which are idiosyncratic features of that particular bargaining procedure. This is problematic because a defining feature of the anarchic international order is that there is no clearly defined institution within which states interact and bargain (Waltz, 1979; Axelrod and Keohane, 1985; Niou and Ordeshook, 1990; Wendt, 1992; Mearsheimer et al., 2001)—in other words, there is no fixed game form. Thus, to explain the relationship between private information and conflict in an environment with multiple related conflict options—the central aim of this paper—the game form should ideally play as small a role as possible. Our game form free approach ensures that our results are robust to the broadest set of possible bargaining procedures that fall within the flexible-response crisis bargaining framework. These robust and generalizable results will spare future scholars the effort of establishing and re-establishing patterns of comparative statics that all flexible-response models will have in common. Instead, future work can focus on alternate facets of flexible-response crisis bargaining models, or on settings where there is a substantive reason to focus on a particular set of protocols.

Our primary contribution is a general characterization of the relationship between private war payoffs, the likelihood of conflict, and equilibrium payoffs in flexible-response crisis bargaining games. We establish, based on model primitives and (at times) a partial equilibrium analysis, when the relationship between a greater private willingness to go to war and a greater likelihood of war is positive and when it is negative. At best—when we know the basic
properties of the defender’s war payoff function and hassling cost function—we can identify the direction of the relationship between private type and war likelihood for any equilibrium in any flexible-response crisis bargaining game. For a broad set of other cases, we can still identify the direction of the relationship, but we may also need to partially characterize equilibrium behavior. In this larger set of cases, we can also identify when there is a positive relationship or a non-monotonic relationship between a greater private willingness to go to war and the defender’s utility.

Building on our formal analysis, we also offer new and substantive insights into the value of specific military technological advancements. Existing research examines the value of certain coercive capabilities within specific military contexts—for example, the value of precision strike capabilities in low-level conflicts and war (Pape, 1996; Horowitz and Reiter, 2001; Kreps and Fuhrmann, 2011; Allen and Martinez Machain, 2019). Most, though not all,4 of the previous research in this area considers only how these technologies fare within an active conflict or after a challenger has already transgressed. However, it is also valuable to know how these capabilities shape bargaining outcomes or the probability of war. Our theory can do this. Once we identify whether improved private wartime capabilities improve hassling capabilities or are detrimental to them, then we are able to make strong statements about how changes in wartime capabilities increase or decrease utilities or the likelihood of war. Armed with our theory, future scholars can leverage existing empirical and public policy research to identify how improving specific capabilities (or a state’s willingness to deploy them) can affect the outcomes in any setting where a flexible-response crisis bargaining model can apply. We also offer further formal advancements: we identify when “always peaceful” equilibria can arise in flexible-response crisis bargaining models, how hassling capabilities specifically shape final settlement utilities, and when to expect more (or less) hassling.

This paper is most similar to Schram (2021b), which considers a deterrence model with multiple conflict options and a publicly observed type that systematically determines conflict utilities. However, the class of models analyzed here generalize that in Schram (2021b); while Schram (2021b) does include a private type, the private type affects only hassling costs rather than affecting both hassling and war costs. This paper, in contrast, offers a model that takes into account both hassling and war costs, and the findings here for flexible-response crisis bargaining games that break the Banks (1990) and Fey and Ramsay (2011) monotonicity results are novel.5 This paper is also similar to a range of research within

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4See Post (2019) as an exception.
5While Schram (2021b) also conducts a game-form free analysis of private type, this analysis finds that a greater private willingness to hassle (or lower private costs to hassling) always results in weakly greater utilities. This is different from the U-shaped relationship between private type and utilities that can arise
the crisis bargaining framework or within the deterrence literature that assume that actors have multiple possible actions available to respond to a crisis (Schultz, 2010; Powell, 2015; McCormack and Pascoe, 2017; Coe, 2018; Spaniel and Malone, 2019; Schram, 2021a; Baliga et al., 2020). Notably, however, this is the first paper to systematically examine how the spillover effects of improvements in one kind of conflict capability can also affect other conflict capabilities, allowing us to offer novel insights into a wide range of previously unconsidered substantive settings. Methodologically, our paper is also similar to a class of work on political science topics that embraces the tools of mechanism design to establish relationships between private information and outcomes. Outside of sources listed above, mechanism design has been featured in work on crisis bargaining and arbitration (Fey and Ramsay, 2009; Hörner et al., 2015; Fey and Kenkel, 2019; Liu, 2021), bureaucracies and delegation (Ashworth and Sasso, 2019), firm regulation (Baron and Besanko, 1987, 1992), legislation and policy-making (Meirowitz et al., 2006), voting (Aghion and Jackson, 2016), and many others.

While this paper uses the language of international relations—a field in which crisis bargaining models have been extensively considered—the model it offers can be applied to any bargaining or negotiating between actors who possess multiple outside options. For example, when insurgent groups or drug cartels compete, they will regularly engage in restrained and precise violent acts (Shapiro, 2013; Schram, 2019; Cruz and Durán-Martínez, 2016); government strategies of repression (Ritter, 2014) or economic extraction (Acemoglu and Robinson, 2012) may constitute a kind of “hassling” against their citizens; and firms engage in a range of conflict-type behaviors with rivals, including price wars, lawsuits, predatory hiring, and others.

1 What do Flexible-Response Crisis Bargaining Models Describe?

To understand flexible-response crisis bargaining models, it will help to start with an example. In 2006, Israel discovered that Syria was building a nuclear reactor. Internally, Israeli decision-makers viewed the possibility of a nuclear-armed Syria as an “existential threat” to the Israeli state (Opall-Rome, 2018). In the context of the Banks (1990) and Fey and Ramsay (2011) monotonicity results, where this high private willingness to go to war leads to a greater likelihood of war, this was the exact setting in which we might expect Israel to go to war. But instead of going to war, Israel used an electronic warfare attack to disable Syrian air defenses, then conducted an airstrike on the reactor (Katz, 2010). Israel’s
airstrike, known as Operation Outside the Box, successfully destroyed a critical component of the Syrian nuclear program and negated the need for a more expansive response.

Flexible response crisis bargaining models can capture political interactions like those surrounding the Syrian reactor development. These models formalize the following interaction. First, a challenger—State C—undertakes some opportunistic and costly action, which we will refer to as a “transgression,” against a defender, State D. Transgressions are politically beneficial to State C but are detrimental to State D. In our example, Syria’s 2008 construction of the nuclear reactor was a transgression that could have eventually led to Syria possessing a nuclear bomb, thus strengthening Syria’s future leverage against Israel. Transgressions like this have been proposed in various fields of scholarship, including work examining enforcement problems in bargaining (Schultz, 2010), deterrence (Fearon, 1997; Gurantz and Hirsch, 2017), and endogenous power shifts (Debs and Monteiro, 2014). Examples of transgressions include states investing in conventional, nuclear, space, or cyber military technologies (Debs and Monteiro, 2014; Gartzke and Lindsay, 2017; Spaniel, 2019), forming alliances (Benson and Smith, 2020), or securing geopolitically valuable territory (Fearon, 1996; Powell, 2006).

In response to the transgression, the defender—State D—has a choice between accepting the transgression through a peaceful settlement, going to war to decisively resolve the political issues between the states, or (as we discuss here) engaging in some low-level actions that undercut the transgression. The first two options—a peaceful settlement or war—are the two standard outcomes used by crisis bargaining or deterrence frameworks. We will refer to the the last option, the low-level response from State D, as “hassling.” As originally defined in Schram (2021a), hassling is the use of limited conflict to degrade a challenger’s rise. Our use of the term here is consistent with this definition, but expands it to include any actions by the defender that undercut the challenger’s transgression. In the Syria case, Israel detected Syria’s nuclear reactor and destroyed it. This is a form of hassling because it was a destructive blow to Syria’s nuclear program, but it was not a decisive military move that would prevent the Assad regime from ever possessing a nuclear weapon. To offer an example of a decisive military move, the 2003 U.S. invasion of Iraq decisively prevented the Ba’athist regime from attaining nuclear weapons by overthrowing it, which is consistent with this model’s treatment of war. Hassling can take the form of limited airstrikes (Reiter, 2005; Fuhrmann and Kreps, 2010), hybrid conflict (Lanoszka, 2016), aspects of gray-zone conflict (Mazarr, 2015; Gannon et al., 2020), (limited) preventive war (Levy, 2011), fait accompli

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6The “transgression” here is similar to the challenger’s first move in a deterrence game in the larger literature; see Huth (1999) for a review of early work on deterrence, as well as Kydd and McManus (2017); Smith (1998); Di Lonardo and Tyson (2017); Chassang and Miquel (2010); Baliga and Sjöström (2020).
(Tarar, 2016), sanctions (McCormack and Pascoe, 2017), or arming (Coe, 2018).

We assume that D possesses a private type that influences both war and hassling capabilities. To offer perhaps the simplest example, suppose a state (such as Israel) is privately very hawkish on the matter of its neighbors developing nuclear weapons. Based on these private preferences, this state might be very willing to conduct a war—and also willing to implement a low-level hassling attack to weaken a target. Ultimately, the question of whether better private capabilities or a greater willingness to go to war also makes low-level options more or less appealing is an empirical one; we discuss substantive applications in section 5.

2 The Flexible Response Model

Here we present the flexible-response crisis bargaining framework. In this framework, negotiations may end efficiently in peace or inefficiently, either in war or in one of a continuum of possible costly non-war outcomes. We assume that a state’s private information may affect its payoffs from both kinds of inefficient outcomes, and we use the tools of Bayesian mechanism design to obtain general results about the relationship between private type and the equilibrium properties.

2.1 Structure of the Interaction

At the outset of the interaction, Nature assigns D’s private type, $\theta \in \mathbb{R}$. Without loss of generality, we assume that D’s war payoff increases with D’s type, while D’s cost of hassling may increase, decrease, or neither. The realized value of $\theta$ is known only to D, but the prior distribution from which it is drawn is common knowledge. Let $F$ denote the CDF of this prior distribution, and let $\Theta$ denote its support.

The interaction between the states takes the familiar form of a crisis bargaining game, except each state may engage in activity that affects outcomes short of war. First, C selects transgression $t \in T \subseteq \mathbb{R}_+$. In response to this transgression, C and D partake in a bargaining process that may end in war or in some hassling response by D. Like Banks (1990) and Fey and Ramsay (2011), we place no particular structure on the bargaining process. We simply assume that each player chooses from a set of available bargaining actions; these choices determine whether the game ends in war, and, if not, how the prize is divided. In

\footnote{In terms of the traditional “costly lottery” formulation of crisis bargaining games, greater $\theta$ may correspond to a greater probability of winning, a lower cost of fighting, or both. However, we do not impose a costly lottery model—all that matters for our purposes is that D’s expected utility from war strictly increases with $\theta$.}
the negotiation stage, we let $b_C \in B_C$ denote $C$’s bargaining strategy (offers, counteroffers, accept-reject plans, etc.). D’s strategy consists of an analogous bargaining strategy $b_D \in B_D$, as well as a level of hassling, $h \in \mathcal{H} \subseteq \mathbb{R}_+$. A game form $G$ consists of the bargaining action spaces, $\mathcal{B}_A$ and $\mathcal{B}_D$, along with an outcome function $g$ that maps the choices $(t, h, b_C, b_D)$ into the set of possible crisis bargaining outcomes. We decompose the outcome function $g$ into three components: whether war occurs, what $C$ receives from bargaining, and what $D$ receives from bargaining. Whether war occurs or not depends solely on actions taken in bargaining. Let $\pi^g(b_C, b_D) \in \{0, 1\}$ be an indicator for whether the interaction ends without war. Conditional on war not occurring, each player’s payoff depends on the bargaining behavior, $C$’s choice of transgression, and D’s selection of hassling. Let $V_C^g(t, h, b_C, b_D)$ and $V_D^g(t, h, b_C, b_D)$ denote the benefits that $C$ and $D$ receive, respectively, in case war is avoided.

D’s private type and the selected transgression and hassling levels shape the final utilities in the game. If war occurs, war payoffs depend on D’s private information, but they do not depend on any of the endogenous choices in the game, including transgressions and hassling. We therefore write war payoffs as $W_C(\theta)$ and $W_D(\theta)$. We assume $W_D$ is strictly increasing, so higher types of $D$ can be interpreted as stronger in wartime. If war is avoided, each player receives their division of the spoils but must pay the cost of their transgression or hassling. Let $K_C(t, h)$ denote the cost to $C$, and let $K_D(h, \theta)$ denote the cost to $D$. We assume that $K_C$ is strictly increasing in $t$ and weakly decreasing in $h$, and we assume that $K_D$ is strictly increasing in $h$. For now, we are agnostic whether $K_D$ is increasing or decreasing in $\theta$. We let $h = 0$ denote no hassling, which entails assuming that $0 \in \mathcal{H}$ and $K_D(0, \theta) = 0$ for all $\theta$. Putting these together, the players’ utility functions in a given game form are as follows:

\begin{align}
  u_D^g(t, h, b_C, b_D | \theta) &= (1 - \pi^g(b_C, b_D)) W_D(\theta) + \pi^g(b_C, b_D) [V_C^g(t, h, b_C, b_D) - K_C(t, h)], \\
  u_D^g(t, h, b_C, b_D | \theta) &= (1 - \pi^g(b_C, b_D)) W_D(\theta) + \pi^g(b_C, b_D) [V_D^g(t, h, b_C, b_D) - K_D(h, \theta)].
\end{align}

Footnotes:

8. We place no restriction on whether hassling is chosen before, during, or after the bargaining process—all that matters for our purposes is that the cost of any given level $h \in \mathcal{H}$ is independent of $b_C$ and $b_D$.

9. The game form represents the elements of the model that are specific to a particular bargaining protocol. Implicitly, then, we take the type space, prior distribution, transgression action set, hassling action set, cost functions, and war payoff functions as primitives of the model rather than features of a game form $G$.

10. Unlike Banks (1990) and Fey and Ramsay (2011), we allow for inefficient settlements. This means that D’s value from bargaining cannot be immediately deduced from C’s, and vice versa.

11. By ruling out $\pi^g \in (0, 1)$, we are implicitly assuming that the bargaining process has no exogenous random components (see Fey and Kenkel, 2019).

12. We relax this in an extension below, in which transgressions and hassling may affect war payoffs.

13. We restrict attention to models in which the players’ payoffs from any peaceful outcome do not depend on D’s private information, except insofar as that private information affects the cost to D of the chosen hassling level $h$. 

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We restrict our attention to game forms in which neither player can force a settlement on the other. This assumption reflects the anarchic nature of international politics, in which states always have the option to resort to force if desired. A sufficient condition is that each player has an action \( b_i \in B_i \) such that \( \pi^g(b_i, b_j) = 0 \) (war is guaranteed) for all \( b_j \in B_j \). As we show below, this condition places important limits on what kinds of outcomes are sustainable as equilibria.

The relationship between D’s private type and hassling ability is central to our analysis. We will show that the effects of \( \theta \) on equilibrium outcomes depend critically on whether greater payoffs from war are associated with higher or lower costs of hassling. In general, when comparing types \( \theta' \) and \( \theta'' \), we say that \( \theta'' \) has greater hassling effectiveness than \( \theta' \) if \( K_D(h, \theta'') < K_D(h, \theta') \) for all \( h > 0 \). We focus on cases where there is a monotone relationship between private type and hassling effectiveness. We say that \( \theta \) improves hassling effectiveness if \( K_D(h, \theta'') < K_D(h, \theta') \) for all \( h > 0 \) and \( \theta' < \theta'' \). In the opposite case, when \( K_D \) strictly decreases with D’s type, we say \( \theta \) degrades hassling effectiveness.

In this class of games, hassling effectiveness concerns the relationship between D’s type and the absolute cost of hassling. When we relate D’s type to the marginal cost of hassling, we obtain even stronger results. We say the hassling cost function satisfies decreasing differences if types with lower absolute costs also have lower marginal costs. This condition ensures that D’s settlement utility has the single-crossing property, allowing us to characterize monotone comparative statics without imposing specific functional forms (Ashworth and Bueno de Mesquita, 2006).

**Definition 1.** The cost function \( K_D \) has decreasing differences in \( h \) and \( \theta \) if

\[
\theta' \text{ has greater hassling effectiveness than } \theta \Rightarrow K_D(h', \theta') - K_D(h, \theta') < K_D(h', \theta) - K_D(h, \theta) \text{ for all } h < h'.
\]

(DD)

### 2.2 Solution Concept and Direct Mechanisms

We restrict attention to pure strategy perfect Bayesian equilibria of each flexible-response crisis bargaining game. Depending on the bargaining protocol and the equilibrium selected, the equilibrium path may be very complex, involving numerous offers and counteroffers before concluding, or it may be simple, ending quickly in war or a settlement. We will not dwell on the details of bargaining itself, as our primary concern is the outcome of the interaction: whether war prevails, and if not, what each party receives from a bargained outcome.
We will focus on the incentives of D, the player with private information. Given an equilibrium of a flexible-response crisis bargaining game, we can summarize the outcome of the game for each type of D with three functions:\(^{14}\)

- Their hassling level, \(h(\theta)\).
- Whether a bargained outcome prevails, \(\pi(\theta)\).
- Their settlement value in case of a bargained resolution, \(V_D(\theta)\).

A direct mechanism for D consists of these functions, \((h, \pi, V_D)\). If type \(\theta\) of D were to follow the equilibrium bargaining strategy of type \(\theta'\), D’s expected utility from doing so would be:

\[
\Phi_D(\theta' | \theta) = (1 - \pi(\theta'))W_D(\theta) + \pi(\theta')[V_D(\theta') - K_D(h(\theta'), \theta)].
\]

While mimicking another type’s strategy may change the hassling level, the occurrence of conflict, and the settlement value for \(\theta\), it does not change D’s war payoff, nor the cost D pays for any given hassling level.\(^{15}\) The key requirement of Bayesian equilibrium is that no type can increase its payoff by mimicking another type’s bargaining strategy. We can phrase this requirement as an incentive compatibility condition on the direct mechanism. Let \(U_D(\theta)\) denote each type’s expected utility along the path of play, so that \(U_D(\theta) = \Phi_D(\theta | \theta)\).

**Definition 2.** A direct mechanism \((h, \pi, V_D)\) is incentive compatible if

\[
U_D(\theta) \geq \Phi_D(\theta' | \theta) \quad \text{for all } \theta, \theta' \in \Theta.
\]

To identify regularities in the outcomes of flexible-response crisis bargaining games, we will analyze incentive-compatible direct mechanisms. We rely on the revelation principle: for any Bayesian Nash equilibrium of a particular game form, there is an incentive-compatible direct mechanism that yields the same outcome (Myerson, 1979). Logically, this means that if we find that some property holds for all incentive-compatible direct mechanisms, than it is true of all equilibria of all flexible-response crisis bargaining games. Without bogging

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\(^{14}\)In the Appendix, we formally define an equilibrium and describe how a direct mechanism can be derived from it. See the discussion in Fey and Ramsay (2011).

\(^{15}\)The definition of \(\Phi_D\) illustrates an important difference between the flexible-response framework and the environment studied by Fey and Ramsay (2009). While Fey and Ramsay (2009) also allow for inefficient bargained settlements, in their framework, the inefficiency loss due to reporting \(\theta'\) is the same for all types \(\theta\). In contrast, we assume the cost of hassling is a function of the true type \(\theta\), which implies that different types may value the “same” settlement differently.
ourselves down in the particulars of how crisis bargaining plays out in any particular game, we are able to characterize robust properties of the outcomes of any flexible-response crisis bargaining game.

Recall that we only consider game forms in which neither player can impose a settlement on the other. This condition ensures that no type of D may receive less than its war payoff in equilibrium—if a settlement would yield less, then it would be profitable for D to deviate to fighting. In the language of mechanism design, this requirement amounts to a participation constraint, or what Fey and Ramsay (2011) call voluntary agreements in the crisis bargaining context.

**Definition 3.** A direct mechanism \((h, \pi, V_D)\) has **voluntary agreements** if

\[
\pi(\theta)[V_D(\theta) - K_D(h(\theta), \theta)] \geq \pi(\theta)W_D(\theta) \quad \text{for all } \theta \in \Theta. \tag{VA}
\]

Naturally, the voluntary agreements condition is automatically satisfied for those types that fight in equilibrium. The constraint only applies to the types that settle—the settlement must yield at least as much as their war payoff, even when accounting for the costs of the hassling. Throughout the analysis, we will restrict attention to direct mechanisms that satisfy both (IC) and (VA), as any equilibrium of a flexible-response crisis bargaining game with voluntary agreements must be outcome-equivalent to some such mechanism (Fey and Ramsay, 2011).

### 3 Private Type and the Probability of War

In crisis bargaining games without flexible responses, private signals of high strength or resolve are associated with a greater equilibrium probability of war (Powell, 1989; Morrow, 1989; Fearon, 1995; Schultz, 1999; Wittman, 2009; Slantchev and Tarar, 2011; Benson et al., 2016; Tarar, 2021). A simple intuition drives this result: if some type finds it worthwhile to run a high risk of war to receive a better deal at the bargaining table, then all stronger types must be willing to run at least as great a risk. For stronger types, the benefits from any settlement are the same, but their payoffs from the fail case of fighting are better.

For equilibria with no hassling, our model recovers the classic monotone relationship between private type and the likelihood of conflict. As long as \(h(\theta) = 0\) for all types, stronger types never have a lower probability of conflict than weaker types. The following result recovers
Figure 3: The relationship between type \( \theta' < \theta'' \) and war likelihood in all flexible-response crisis bargaining games.

Lemma 1 of Banks (1990) as a special case in our environment.\(^{16}\)

**Lemma 1.** If \( h = 0 \) and \( \theta' < \theta'' \), then \( \pi(\theta') \geq \pi(\theta'') \).

This result confirms that the exceptions we find to the classic model are due exclusively to our introduction of flexible responses, not to any other feature of our modeling environment. However, once states begin to employ alternative instruments for altering the balance of bargaining power, this straightforward relationship between private strength and the risk of war holds only under special conditions.

The flow chart in Figure 3 summarizes our findings on the relationship between private information and the occurrence of conflict. If private strength degrades hassling effectiveness, then stronger types are more likely to fight, just as in traditional crisis bargaining models. The same is true if private strength increases hassling effectiveness and has a stronger effect on war payoffs than on the cost of hassling (WURI condition, formally defined below). However, we find that weaker types are more likely to fight—the opposite of the traditional result—when private strength improves hassling effectiveness but affects war payoffs less than hassling costs (SURI, also defined below). If there is no monotone relationship between private strength and the cost of hassling, or if, when stronger types are more effective, neither of the aforementioned conditions on the relative effects of private type hold, then we cannot pin down a clear relationship with the probability of conflict.

\(^{16}\)Though we restrict to deterministic outcomes and thus \( \pi(\theta) \in \{0,1\} \) in the bulk of our analysis, the proof of Lemma 1 holds even if we allow for probabilistic outcomes. All proofs are in the Appendix.
When hassling takes place, the effect of private type on the chance of war depends on whether a state’s private war capability improves or degrades its hassling effectiveness. If private type degrades hassling effectiveness—i.e., if types with greater war payoffs also have greater costs of hassling—then higher types are weakly more likely to go to conflict.

Proposition 1. Assume $\theta$ degrades hassling effectiveness. If $\theta' < \theta''$, then $\pi(\theta') \geq \pi(\theta'')$.

Because the value of an efficient settlement without hassling is the same regardless of $\theta$, higher types have the most incentive to choose war over an efficient settlement. If private type degrades hassling effectiveness, then this logic carries over to settlements involving hassling as well. If some type of D prefers war over a settlement with hassling level $h \geq 0$, then all stronger types must have the same preference: they have an even higher war payoff and would pay a greater cost from the same level of hassling.

When private type is instead associated with lower costs of hassling, we need more conditions to characterize its relationship with the equilibrium occurrence of war. In this case, not only is war more attractive to stronger types of D, but so is any given settlement with hassling. Because war payoffs and settlement payoffs are now moving in the same direction as a function of D’s type, the critical question for our purposes is which rate of increase is quicker. The probability of conflict increases with D’s private strength in equilibrium if $\theta$ has a stronger effect on war payoffs than on the cost of hassling. On the other hand, if the effect of $\theta$ on hassling cost is dominant, then weaker types are more likely to fight in equilibrium—the opposite of the pattern found in traditional crisis bargaining games.

To prove these claims, we must formally state what it means for $\theta$ to affect war payoffs more than the cost of hassling, or vice versa. We say the war utility is relatively increasing (WURI) when $\theta$ has a greater marginal effect on war payoffs than on the cost of hassling. In the opposite case, we say the settlement utility is relatively increasing (SURI).

Definition 4. In a direct mechanism, the war utility is relatively increasing if

$$W_D(\theta'') - W_D(\theta') > K_D(h(\theta''), \theta') - K_D(h(\theta''), \theta'')$$

for all $\theta', \theta'' \in \Theta$ such that $\theta' < \theta''$ and $\pi(\theta'') = 1$. 

(WURI)
The *settlement utility is relatively increasing* if

\[
W_D(\theta'') - W_D(\theta') < K_D(h(\theta'), \theta') - K_D(h(\theta'), \theta''),
\]

for all \( \theta', \theta'' \in \Theta \) such that \( \theta' < \theta'' \) and \( \pi(\theta') = 1 \).

(SURI)

If either of these holds and private strength increases hassling effectiveness, then we can identify a monotonic relationship between private information and the equilibrium chance of conflict for any flexible-response crisis bargaining model.

**Proposition 2.** Assume \( \theta \) improves hassling effectiveness, and let \( \theta' < \theta'' \). If (WURI) holds, then \( \pi(\theta') \geq \pi(\theta'') \). If (SURI) holds, then \( \pi(\theta') \leq \pi(\theta'') \).

This result shows that the conventional relationship between private information and the likelihood of conflict is not robust to the introduction of hassling that affects payoffs from bargaining. Assuming that types with greater battlefield effectiveness are also more effective at hassling activities, the relationship between \( \theta \) and the likelihood of conflict depends critically on the technology of hassling. If the marginal effect of D’s type on the costs of hassling always outweighs its effect on the war payoffs, then we have the opposite of the usual result, with stronger types less likely to fight on the path of play.

The two conditions we have outlined here are mutually exclusive (except in the trivial case where all types end up fighting in equilibrium), but they are not mutually exhaustive. Depending on the functional forms of \( W_D \) and \( K_D \), it is possible for the marginal effect of \( \theta \) on the war payoff to be relatively strong for some types and relatively weak for others. In this scenario, we cannot generally characterize the relationship between D’s private type and which outcome prevails in equilibrium.

While Proposition 2 is useful for understanding how private information affects the occurrence of war in flexible response crisis bargaining games, its practical applicability is somewhat limited. Ideally, we would be able to say on the basis of the model primitives—the war payoff and hassling cost functions—whether stronger types will be associated with a greater likelihood of conflict in any given strategic environment. However, the WURI and SURI conditions do not exclusively concern model primitives, as they depend on the levels of hassling chosen on the path of play. This raises the possibility that the relationship between D’s private type and the likelihood of conflict may vary depending on the exact bargaining protocol.
With additional conditions on the model primitives, we can ensure that the war utility is relatively increasing, meaning that the likelihood of conflict increases with D’s type. In particular, we need the cost function to have decreasing differences and the marginal effect of θ on the war payoff to always exceed its marginal effect on the hassling cost when h is at its upper bound. Under these conditions, higher types are more likely to end up at conflict in the equilibria of all flexible-response crisis bargaining games, regardless of the exact negotiating protocol employed.

**Lemma 2.** Assume θ improves hassling effectiveness, (DD) holds, and max \( H = \bar{h} < \infty \). If

\[
W_D(\theta'') - W_D(\theta') > K_D(\bar{h}, \theta') - K_D(\bar{h}, \theta'')
\]

for all \( \theta', \theta'' \in \Theta \) such that \( \theta' < \theta'' \), then (WURI) holds.

There is not an analogous sufficient condition for the settlement utility to be relatively increasing. The obstacle here is our assumption that \( h = 0 \) is always feasible at zero cost.\(^{17}\) This means the marginal effect of θ on the cost of the hassling is zero for \( h = 0 \), ruling out any kind of sufficient condition for the marginal effect of θ on the hassling cost to always exceed its effect on the war payoff. At most, if we assume decreasing differences in the cost of hassling, we can make the SURI condition slightly less onerous to check. In this case, letting \( \bar{h} \) denote the minimal hassling among types that end up in a bargained resolution, a sufficient condition is that

\[
W_D(\theta') - W_D(\theta) < K_D(\bar{h}, \theta) - K_D(\bar{h}, \theta')
\]

for all \( \theta < \theta' \). If this condition holds, then, in equilibrium, it can never be the case that low types settle and high types go to war.

### 4 Private Type and Payoffs

A second key regularity in crisis bargaining games that do not include flexible responses is that a better private war capability corresponds to a better equilibrium payoff, even when war does not occur (Banks, 1990; Fey and Ramsay, 2011). In these models, greater private strength increases an actor’s willingness to run the risk of war, and risky bargaining actions thus serve as signals of strength, allowing the actor to extract more from negotiations if the interaction does not ultimately end in war.

We recover the same relationship between private strength and equilibrium payoffs in the flexible response environment when hassling does not take place along the path of play. The

\(^{17}\)The normalization of the cost to zero is immaterial, but the constancy of the cost of across types of D is important here.
Does $\theta$ increase hassling effectiveness?

Yes

Stronger types better off:
$U_D(\theta') \leq U_D(\theta'')$

No

Weaker types better off:
$U_D(\theta') \geq U_D(\theta'')$

Does $\theta$ degrade hassling effectiveness?

Yes

Do both types fight?

No

Do both types settle?

Yes

Further analysis needed

Figure 4: The relationship between type ($\theta' < \theta''$) and equilibrium payoffs in all flexible-response crisis bargaining games.

The following result is our analogue of Lemma 4 from Banks (1990).

**Lemma 3.** If $h = 0$ and $\theta' < \theta''$, then $U_D(\theta') \leq U_D(\theta'')$.

Once we introduce additional ways to alter the balance of spoils, the tight relationship between private war capability and equilibrium payoffs no longer holds. The flowchart in Figure 4 summarizes the relationship between private strength and utility in flexible-response crisis bargaining games. Obviously, among types that end up fighting in equilibrium, stronger types are always better off. Outside of that case, however, private strength is only guaranteed to increase payoffs when it is also associated with greater hassling capability. If private strength instead degrades hassling capability, we find the opposite of the usual result: types with lower private strength have greater payoffs in case of peace.

We recover the positive relationship between private strength and equilibrium payoffs when $\theta$ improves hassling effectiveness. In this case, stronger types have an advantage in both channels of bargaining leverage—hassling and the threat of war—and therefore never come away worse off at the bargaining table. In fact, a stronger type has a strictly greater payoff than all weaker types whenever it goes to war in equilibrium or engages in non-zero hassling.

**Proposition 3.** Assume $\theta$ improves hassling effectiveness. If $\theta' < \theta''$, then $U_D(\theta') \leq U_D(\theta'')$.

The inequality is strict if $\pi(\theta') = 0$ or $h(\theta') > 0$. 

19
A simple incentive compatibility logic is behind this result. Consider the outcomes for two types of $D$, one weaker and one stronger. If the weaker one goes to war, then obviously the stronger one could do better by going to war as well. Conversely, if the weaker one settles, the stronger type could get the same terms of settlement as the weaker one by choosing the same hassling level and bargaining actions. Moreover, under the condition that private type improves hassling capabilities, the stronger type’s hassling cost would be no greater than that of the weaker type; in fact, it would be strictly less if $h > 0$. No matter what the outcome for the weaker type, the stronger type has a bargaining strategy available that results in a weakly better payoff. Consequently, the stronger type’s equilibrium payoff must be no lower than the weaker one’s.

If private strength is instead associated with lower hassling effectiveness, then we find an exception to the traditional positive relationship between private type and equilibrium payoff. In this case, the relationship is U-shaped. Low types, which have poor battlefield effectiveness but relatively low costs of hassling, choose to settle rather than to fight in equilibrium. Among these types, lower private strength is associated with greater hassling ability, and thus a greater equilibrium payoff. At a certain level of strength, however, it becomes profitable to fight rather than settle. After this point, greater military strength by necessity leads to a greater payoff.

**Proposition 4.** Assume $\theta$ degrades hassling effectiveness. There exists $\hat{\theta}$ such that $\pi(\theta) = 1$ for all $\theta < \hat{\theta}$ and $\pi(\theta) = 0$ for all $\theta > \hat{\theta}$. If $\theta' < \theta'' < \hat{\theta}$, then $U_D(\theta') \geq U_D(\theta'')$ (strictly if $h(\theta'') > 0$). If $\hat{\theta} < \theta' < \theta''$, then $U_D(\theta') < U_D(\theta'')$.

This result illustrates the new sources of bargaining leverage that arise in crisis bargaining games with flexible responses. In ordinary crisis bargaining games, a state’s sole source of bargaining power is its threat to resort to war. In our framework, hassling provides another means of shifting the peacetime balance of spoils. There is a direct effect of hassling efficiency, where more effective types end up better off because they can afford to hassle more, thereby shifting the balance of goods in their favor. There is also an indirect effect: more effective types can pay a lower cost to undertake the same amount of hassling. Both effects contribute to the negative relationship between private strength and payoffs from peace when $\theta$ degrades hassling effectiveness.

In analyzing private strength’s relationship to both the probability of war and equilibrium payoffs in flexible-response crisis bargaining games, we have identified some conditions under which our results diverge with the traditional patterns and other conditions under which
they agree. Interestingly, we find only one case in which both the chance of conflict and the equilibrium utility increase with private strength, as in the traditional pattern—namely, when $\theta$ increases hassling effectiveness but has an even stronger effect on war payoffs (WURI). The ordinary crisis bargaining model may, in a sense, be considered a special case of these more general conditions. If private strength increases hassling effectiveness while having a relatively low effect on war payoffs (SURI), then equilibrium payoffs increase with $\theta$ as in traditional models, but the probability of war decreases. We see the converse pattern if private strength degrades hassling effectiveness: the probability of war increases with $\theta$ as usual, but the equilibrium utility is U-shaped.

## 5 Empirical Implications

How does our theory update our understanding of conflict? Consider the ongoing debate over the usefulness of aerial bombing in conflict. To quickly summarize a subset of findings, existing research has shown that aerial bombings or precision strike capabilities can be useful in hassling operations (Kreps and Fuhrmann, 2011) or in a conventional war (Pape, 1996; Horowitz and Reiter, 2001; Allen and Martinez Machain, 2019), but may be less effective in a counterinsurgency (Lyall, 2013; Dell and Querubin, 2018). While this research agenda is quite deep, the existing literature does not speak to, for example, what precision strike technology means for shaping deterrence or bargaining.\(^{18}\) In every paper cited above, the value of these weapons was evaluated conditional upon a bargaining failure having occurred. It would be valuable to know how these weapons influence decision making in the lead-up to crises, whether they lead to more (or less) war, and whether they produce expected windfalls for the state that developed the bombing capabilities. These questions are difficult to answer empirically; identifying or constructing a counterfactual world where a state did not develop precision strike capabilities is challenging, as is identifying how these capabilities (or their absences) shape international crisis initiation or behavior. Our theory is well suited to address these questions. The flowcharts in Figures 1 and 2 demonstrate that once we identify whether improved private wartime capabilities improve or are detrimental to hassling capabilities, then we are able to make strong statements about how changes in wartime capabilities increase or decrease the likelihood of war and players’ payoffs from settling.

As an illustration, consider a crisis where (a) a defender state possesses private information about its ability to conduct precision strikes, (b) the most feasible low-level conflict option is a precision strike (along the lines of Operation Desert Fox), and (c) the war option would

\(^{18}\)One notable exception is Post (2019), which analyzes airpower events (i.e. military exercises, mobilizations, shows of force, deployments of military assets, or other military signal) as signals for use in compellence.
be a conventional war (along the lines of Operation Desert Storm). Because precision strike technology is dual-use for both hassling and war, then these private, advanced precision strike capabilities would have a positive effect on both war and hassling outcomes (noted by " + ") symbols in Table 1). Thus, if (a)-(c) held, as the flowcharts in figures 1 and 2 show, any flexible-response crisis bargaining model of this case would find that a defender with a better private ability to conduct precision strikes would end up with greater final payoffs, but not necessarily a greater likelihood of engaging in a conventional war. On the other hand, consider a crisis where (a) and (b) held, but, instead of (c), it was the case that (d) the war option would be a protracted counterinsurgency, where precision strike capabilities would likely be less effective (" - " symbol in Table 1).19 If (a), (b), and (d) held, then any flexible-response crisis bargaining model of this case would find that a defender state with a better private ability (or private willingness) to conduct precision strikes would be less likely to enter into a war, and might or might not end up with greater final expected payoffs.20

A number of other conflict capabilities can be dual-use for hassling and war. During Operation Outside the Box (2007), Israel disabled Syrian air defenses with an electronic warfare attack. While the full details of the electronic warfare attack have not been disclosed, any attack that allowed multiple Israeli aircraft to enter Syria and conduct a raid without harassment plausibly could have also been used to conduct a more extensive conventional attack (Katz, 2010). Additionally, cyberattacks have been used as part of a conventional war (Russia-Georgia War, 2008) and as hassling, both independently (Stuxnet and Estonian cyber attacks) and as a part of a cluster of operations against a target state (the NotPetya attacks targeting Ukraine) (Buchanan, 2020; Gannon et al., 2020). Similarly, developments in anti-satellite technologies have opened the possibility for disruption of GPS signals; low-level disruptions could be used for hassling, and more serious disruptions could create problems for modern air and sea warfare (Harrison et al., 2020). While we do not have the space to discuss every dual-use technology case at length, a number of other capabilities positively affect hassling and war in straightforward ways: the continued development of airlift capabilities can facilitate special operations for use in low-level conflict, conventional war, and irregular warfare (Bolkcom, 2007; Pietrucha and Renken, 2019); coordinating with or providing support to violent non-state actors have been used in both hassling and war (Schultz, 2010; Schram, 2021a); and an ability to control the civilian information environment has been used in both hassling operations (Russia in Eastern Ukraine, 2015-on) and in hearts-and-minds

19 Alternatively, in a setting with budget restrictions, it is plausible that investments in precision strike capabilities were made at the expense of counterinsurgency capabilities.

20 While throughout the text we treat θ as improving war capabilities, we can draw the conclusions stated here due to 1.
counterinsurgencies.

Of course, a number of capabilities are less effective in some forms of conflict. Just as precision strike capabilities may not be well suited for a protracted hearts-and-minds COIN campaign (Lyall, 2013; Dell and Querubin, 2018), neither may the (very expensive) electronic, cyber, or anti-satellite tools;\(^{21}\) insofar as states over-invest in precision strike capabilities, those states may be less able to conduct a war requiring extensive COIN operations. Similarly, civilian information operations can help in a hassling or COIN context, but these operations may not be as well-suited for a conventional conflict. And finally, many conventional warfare technologies are unsuited for use in hassling scenarios. For example, the U.S. Navy, Marine Force, and Coast Guard have jointly issued a report suggesting that conventional navy vessels designed for lethality in conventional warfare may not be as effective in deft handling of gray zone attacks in the Pacific theater, saying that a more agile fleet could better serve in “gray zone” operations (Berger \textit{et al.}, 2020; Owen, 2021).

Other factors besides the development of capabilities could shape a leadership’s incentives in such a way that alters their willingness to engage in some forms of conflict over others. Statistical analyses of the "rally-round-the-flag" phenomenon suggest that larger military operations, especially wars, generate an increase in public support for domestic leadership (Baker and Oneal, 2001; Chapman and Reiter, 2004). This effect could alter a leader’s preferences, causing them to prefer war over hassling operations. Alternatively, domestic political economy considerations can also shape leaders’ incentives. For example, in 1954 the United States provided arms, funds, and training to Guatemalan rebels who overthrew Jacobo Árbenz and installed right-wing dictator Castillo Armas—a move that benefited the politically connected United Fruit Company (Kinzer, 2007, 125–147) in ways that sanctions would not. Similarly, while a “blood for oil” hypothesis may not fully explain 2003 Iraq invasion (Paul, 2003; Stokes, 2007), it could have still shaped a private willingness for U.S. leadership to go to war rather than hassle. Of course, this is not to say that relevant domestic actors always prefer war, as the use of international sanctions or tariffs to weaken a regime can also create domestic winners and losers.

\(^{21}\)There are some notable exceptions (see Shachtman, 2011).
<table>
<thead>
<tr>
<th>Capability/Willingness Mechanism</th>
<th>Hassling Type</th>
<th>War Type</th>
<th>Rationale</th>
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<tr>
<td>Precision Strikes/Drones</td>
<td>Targeted strikes against military facilities (+)</td>
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<td>Dual-Use Technology</td>
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<tr>
<td>Electronic/Cyber/Anti-Satellite Attacks</td>
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<td>Airlift capabilities</td>
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<tr>
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<tr>
<td>Domestic Information Operations</td>
<td>Undermining domestic authority/promoting discord (+)</td>
<td>Irregular War (+)</td>
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<tr>
<td>Domestic Information Operations</td>
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<td>Precision, Electronic, Cyber, Anti-Satellite Attacks</td>
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<td>Conventional Navy gray hulls optimized for high-end naval warfighting</td>
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<tr>
<td>Domestic Electoral Considerations</td>
<td>Sanctions, hassling, or any other low-level response (-)</td>
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<tr>
<td>Domestic Political Economy Considerations</td>
<td>Sanctions or low-level conflict (+/-)</td>
<td>Conventional War (+/-)</td>
<td>Political Elite Disconnect</td>
</tr>
</tbody>
</table>

Table 1: How various capabilities or willingness mechanisms affect hassling and war capacities. The “+” symbol (“-” symbol) indicates that the capability or willingness mechanism improves (is detrimental to) the listed hassling operations or war type.
6 Additional Analysis

6.1 Possibility of Peace

Previous game-free analyses of crisis bargaining have sought to identify sufficient conditions for peace to be an equilibrium outcome (Fey and Ramsay, 2009, 2011). The mechanism design approach we use here is well suited for studying the possibility of peace. If a given game form has a positive probability of war in equilibrium, that raises the question of whether states could avoid conflict if they chose a different bargaining protocol. We now ask, once the relationship between private type, war payoffs, and hassling costs is established, whether there is any crisis bargaining game with an equilibrium that would end peacefully. After all, any given game form may result in a positive probability of war, but that does not mean war is inevitable—in the international system, states are not bound to follow any particular bargaining protocol.

When working with flexible-response crisis bargaining games, we must be explicit about what it means for an equilibrium to be peaceful. At a minimum, as in ordinary crisis bargaining games, the game must end with a negotiated settlement for all types $\theta \in \Theta$. Furthermore, because transgressions and hassling may be interpreted as forms of low-level conflict, we will focus on equilibria in which $C$ chooses $t = 0$ and each type of $D$ chooses $h = 0$. Mirroring the terminology of Fey and Ramsay (2011), we will call an equilibrium meeting these conditions always peaceful.

In our baseline flexible-response context, the sufficient condition for peace is virtually the same as in ordinary crisis bargaining models. In particular, it must be possible to divide the pie so as to simultaneously satisfy both $C$ (assuming $C$’s knowledge of $D$’s type is limited to the prior distribution) and the strongest type of $D$. In what follows, let $\hat{W}_C = E[W_C(\theta)] = \int_{\Theta} W_C(\theta) dF(\theta)$, $C$’s prior expectation of its own war payoff.

**Proposition 5.** If $\hat{W}_C + W_D(\bar{\theta}) \leq 1$, then there is a flexible-response crisis bargaining game form with voluntary agreements that has an always peaceful equilibrium.

The condition of Proposition 5 is least likely to hold when the distribution of $D$’s type is right-skewed. In this case, $C$’s expected war payoff will be relatively high, since $D$’s type is likely to be low. It will thus be impossible to satisfy the (rarely occurring) strongest type of $D$ while giving $C$ at least its expected war payoff.

If $C$’s war payoff is independent of $D$’s type (i.e., $D$’s type only affects its cost of fighting, not
its probability of victory), then the condition of Proposition 5 is sure to hold. A distribution of the pie following the probability of war will be acceptable both to C and to all types of D. The following result is a direct analogue of Proposition 2 in Fey and Ramsay (2011).

Corollary 1. If $W_C(\theta) = p - c_C$ and $W_D(\theta) = 1 - p - c_D(\theta)$, where $c_D : \Theta \rightarrow \mathbb{R}_+$, then there is a flexible-response crisis bargaining game form with voluntary agreements that has an always-peaceful equilibrium.

One may wonder why our conditions for always peaceful equilibria do not depend on the costs of transgressions or hassling. This is due to our baseline assumption that $t$ and $h$ can only affect payoffs from negotiations, not from fighting. If the war payoffs were also functions of these choices, then the players’ reservation values for conflict would depend on the marginal effects and costs of transgressions and hassling. In order for the flexible-response framework to materially affect the prospects for peace, responses must shape payoffs in war as well as peace.\(^{22}\)

6.2 Terms of Settlement

In ordinary crisis bargaining games, the only way to get a better deal at the bargaining table is to run a greater risk of war (Banks, 1990, Lemma 3). As we observed in our analysis of private type and equilibrium payoffs above, flexible responses like hassling introduce another mechanism for states to obtain better terms from a settlement. Even with little or no threat of war, a state may use transgressions or hassling to shift the balance of bargaining power.

But because hassling is costly, any increase in hassling must come with a commensurate benefit in the terms of settlement; if a state could obtain a better deal while incurring a lower cost of hassling, it would obviously be a profitable deviation. Consequently, whenever we compare two types that both end up at peace in the equilibrium of a flexible response crisis bargaining game, the one that hassles more must get a better deal. If both hassle the same amount, then they should receive identical settlements—just like states that run the same risk of war in ordinary crisis bargaining games.

Proposition 6. If $\pi(\theta) = \pi(\theta') = 1$ and $h(\theta) \leq h(\theta')$, then $V_D(\theta) \leq V_D(\theta')$. Furthermore, if $h(\theta) < h(\theta')$, then $V_D(\theta) < V_D(\theta')$.

\(^{22}\)A similar mechanism drives results in Liu (2021).
Placing additional structure on the model primitives allows us to be even more specific about the relationship between the extent of hassling and the value of settlement. First, we will assume D’s type is drawn from an interval, $\theta \in [\bar{\theta}, \bar{\theta}]$. This requirement effectively allows us to strengthen the incentive compatibility conditions for equilibrium, as we can now say that every type of D must find it unprofitable to mimic the strategy of a marginally stronger or weaker type. Second, we will assume a certain degree of differentiability (and thus continuity) in the relationship between private type and war payoffs, as well as that between private type, hassling amount, and the cost of hassling. These assumptions allow us to characterize local incentive compatibility conditions—the lack of incentive to mimic a slightly lower or higher type—in terms of derivatives of the war payoff and hassling cost functions. We refer to the collection of these assumptions as bounded variation conditions, or (BV).

**Definition 5.** The model has *bounded variation* if $W_D$ and $K_D$ are differentiable,

$$\begin{align*}
\Theta &= [\theta, \bar{\theta}] \quad \text{where } \theta < \bar{\theta}, \\
|W_D(\theta) - W_D(\theta')| &\leq M_W|\theta - \theta'| \quad \text{for all } \theta, \theta' \in \Theta, \text{ where } M_W < \infty, \\
|K_D(h, \theta) - K_D(h', \theta')| &\leq M_D\|(h, \theta) - (h', \theta')\| \quad \text{for all } h, h' \in \mathcal{H} \text{ and } \theta, \theta' \in \Theta, \text{ where } M_D < \infty.
\end{align*}
$$

(BV)

The bounded variation conditions allow us to apply the “envelope theorem” commonly employed in mechanism design analyses of crisis bargaining models (Banks, 1990; Fey and Ramsay, 2011). Given just a few endogenous elements of the equilibrium, we can determine every type’s equilibrium payoff, which in turn will allow us to back out the precise terms of settlement for each type that ends up at peace. All we need to know are the lowest type’s equilibrium utility, whether each type ends up at war or peace, and the extent of hassling carried out by those types that end up at peace. The following proposition gives a precise statement of $U_D(\theta)$ as a function of these equilibrium quantities.

**Lemma 4.** Assume (BV) holds. For all $\theta_0 \in \Theta$,

$$U_D(\theta_0) = U_D(\theta) + \int_{\theta}^{\theta_0} (1 - \pi(\theta)) \frac{dW_D(\theta)}{d\theta} d\theta - \int_{\theta}^{\theta_0} \pi(\theta) \frac{\partial K_D(h, \theta)}{\partial \theta} \bigg|_{h = h(\theta)} d\theta. \tag{3}$$

23Specifically, we assume that $W_D$ and $K_D$ are Lipschitz continuous, a weaker requirement than continuous differentiability.

24In fact, all that is necessary is to know the equilibrium payoff of a single type, not necessarily that of $\theta$.  

27
This complex expression boils down to two essential facts about the relationship between private type and equilibrium payoffs. First, for types that go to war, the marginal increase in utility as \( \theta \) increases is the same as the marginal increase in war payoff—naturally enough. Second, among those that settle in equilibrium, the marginal change in equilibrium payoff depends exactly on the marginal effect of private type on the cost of hassling. This second fact is what allows us to pin down the value of settlement once we know which types settle and how much they spend on hassling. Suppose there is an interval of types \( [\theta', \theta''] \subseteq \Theta \) which all choose to settle in equilibrium, or \( \pi(\theta) = 1 \) for all \( \theta \in [\theta', \theta''] \). We can use Lemma 4 to characterize how the terms of the bargain differ between the poles of this interval:

\[
V_D(\theta'') - V_D(\theta') = K_D(h(\theta''), \theta'') - K_D(h(\theta'), \theta') - \left. \int_{\theta'}^{\theta''} \frac{\partial K_D(h, \theta)}{\partial \theta} \bigg|_{h=h(\theta)} d\theta \right.
\]

The first term here, the cost difference, is a kind of baseline: incentive compatibility means the terms of settlement must adjust at least roughly in accordance with the cost paid for hassling. If not for the second term, the effectiveness premium, then each type \( \theta \in [\theta', \theta''] \) would have the same equilibrium payoff. The effectiveness premium represents the additional benefit that states with greater hassling effectiveness can extract from bargaining. For example, suppose \( \theta \) improves hassling capability, or that \( K_D \) is decreasing in \( \theta \). Then the effectiveness term will be positive, more so if there is a steep relationship between private type and the marginal cost of hassling. Conversely, there will be no benefit from the effectiveness premium if hassling does not take place. If \( h(\theta) = 0 \) for all \( \theta \in [\theta', \theta''] \), then the cost difference and effectiveness premium are both zero, and all types in this interval receive the same settlement. In that case, we are back in the world of ordinary crisis bargaining games, where the only source of bargaining leverage is the threat of war.

In the special case of the bounded variation model where \( \theta \) degrades hassling capability, we can further pin down the value of settlement. We know from Proposition 1 that any equilibrium in this case will be characterized by a cutpoint \( \hat{\theta} \in \Theta \), with all types below \( \hat{\theta} \) settling in equilibrium and all types above it fighting. Using Lemma 4, we can then characterize the settlement value for all \( \theta < \hat{\theta} \) in terms of the cutpoint type’s war payoff and the choice of hassling by each intermediate type.

**Corollary 2.** Assume \( \theta \) degrades hassling effectiveness and (BV) holds. There exists \( \hat{\theta} \in \Theta \)
such that \( \pi(\theta) = 1 \) for all \( \theta < \hat{\theta} \) and \( \pi(\theta) = 0 \) for all \( \theta > \hat{\theta} \). If \( \theta < \hat{\theta} < \bar{\theta} \), then for all \( \theta_0 < \hat{\theta} \),

\[
V_D(\theta_0) = W_D(\hat{\theta}) + K_D(h(\theta_0), \theta_0) + \int_{\theta_0}^{\hat{\theta}} \left. \frac{\partial K_D(h, \theta)}{\partial \theta} \right|_{h=h(\theta)} d\theta.
\]

(4)

This result is useful for two reasons. First, it shows that we need relatively little information about the equilibrium to pin down settlement values in flexible-response crisis bargaining games when \( \theta \) degrades hassling capability. As long as we know the lowest type that fights and the amount of hassling exerted by each lower type, we can derive the exact settlement level in equilibrium. Importantly, if two bargaining games result in the same cutpoint type and the same amount of hassling below the cutpoint, they will also result in the exact same terms of settlement for each type of \( D \), even if the bargaining processes themselves are quite dissimilar. Second, as we show in the next section, the necessary condition provided by Corollary 2 turns out to be sufficient. Specifically, for any non-decreasing hassling plan \( h(\theta) \), if we allocate settlement values according to the given formula for \( V_D \), the resulting mechanism is incentive compatible and satisfies voluntary agreements.

6.3 Amount of Hassling

We have shown that greater hassling effectiveness is associated with greater utility in case the crisis ends in a settlement (Proposition 3 and Proposition 4). We have also shown that the only way to improve one’s payoff from a settlement is to choose greater levels of hassling (Proposition 6). Intuitively, then, it would appear to follow that more effective types hassle more in equilibrium. This intuition only holds in general when the cost of hassling satisfies the decreasing differences condition.

**Proposition 7.** Assume (DD) holds. If \( \pi(\theta) = \pi(\theta') = 1 \) and \( \theta' \) has greater hassling effectiveness than \( \theta \), then \( h(\theta) \leq h(\theta') \).

Because we can only say that more effective types hassle weakly more, this result leaves open two possibilities about why exactly more effective types have better equilibrium payoffs. One is that they hassle the same amount and receive the same settlement, so the higher payoff comes solely from the lower cost of hassling. The other possibility is that they hassle more and get better terms of settlement. Only if decreasing differences holds can we rule out a third possibility—that the more effective types hassle slightly less but at much lower cost, for a net increase in payoff despite the decrease in terms of settlement.
Combined with our earlier results on settlement values, Proposition 7 implies that the settlement value $V_D$ increases with hassling effectiveness, as long as the decreasing differences condition holds. If $\theta$ enhances hassling effectiveness, this implies that $V_D$ is weakly increasing on the subset of $\Theta$ where peace prevails. If instead $\theta$ degrades hassling effectiveness, it means that $V_D$ is weakly increasing on $[\theta, \hat{\theta})$, where $\hat{\theta}$ is the lowest type that weakly prefers to fight (see Corollary 2).

So far we have found that incentive compatibility implies that the level of hassling increases with effectiveness, as long as the decreasing differences condition holds. Can we say anything more specific about the relationship between private type and the equilibrium choice of hassling in a broad class of flexible-response crisis bargaining games? If private strength is associated with lower hassling effectiveness, the answer turns out to be no: virtually any weakly decreasing hassling plan (subject to some continuity restrictions) can be sustained as the equilibrium of some bargaining game.

**Proposition 8.** Assume $\theta$ degrades hassling effectiveness and (BV) and (DD) hold. Let $h$ be any non-increasing and absolutely continuous function from $[\theta, \hat{\theta}]$ into $H$. Take any $\hat{\theta} \in (\theta, \bar{\theta})$, let $\pi(\theta) = 1\{\theta \leq \hat{\theta}\}$, and let $V_D(\theta)$ be defined by Equation 4. The direct mechanism $(h, \pi, V_D)$ satisfies (IC) and (VA).

This result demonstrates that incentive compatibility and voluntary agreements alone place no restrictions on the pattern of hassling across types beyond the fact that more effective types cannot engage in less of it. Consequently, the specifics of the relationship between effectiveness and the degree of hassling are highly model-dependent. For example, we cannot determine from the primitives alone whether all types hassle to the same degree, or whether there is some separation in levels of hassling. This will depend on how bargaining takes place and the precise effects of hassling choices on peacetime payoffs.

### 6.4 Limitations

Our framework captures a crisis environment where states have multiple response options: they can form fully peaceful resolutions, conduct costly, low-level revisions to the peaceful resolution, or go to war. While the flexible-response crisis bargaining framework embraces a broader set of possible actions than what is allowed in standard crisis-bargaining models, it also has some subtleties and limits worth expanding on.

Our framework is distinct from what is covered in the Fey and Ramsay (2011) discussion.
of “war as a bargaining process” (pp. 166–167). In the setup in Fey and Ramsay, as in the standard setup in models like Wagner (2000), Filson and Werner (2002), or Slantchev (2003a), there can be inefficiencies within bargained outcomes, and these inefficiencies could be interpreted as allowing for low-level conflict to occur within bargaining. We concede that this interpretation is sometimes correct, but only when low-level conflict follows a very specific progression in which each round or degree of fighting has an identical structure. In other words, in the “war as a bargaining process” setup, the costs of ten rounds of fighting are identical to ten times the costs and benefits of one round of fighting. This symmetry could be appropriate if low-level conflict used the same technology as a decisive war but was (proportionally) smaller or conducted over a shorter period. In most cases, however, this seems like too strong an assumption: sanctions, hassling, hybrid war, and gray zone conflict utilize different technologies and different force postures than conventional war does, suggesting that these capabilities do not share a common underlying cost structure.

We highlight two shortcomings of our framework as avenues for future research. First, the flexible-response crisis bargaining framework does not speak to scenarios where observations of transgressions or hassling decisions are noisy. This undercuts our framework’s ability to describe, for example, settings where a transgression is imperfectly observed or where identifying attribution is hard. We acknowledge that our framework is not well suited to describe cyberwarfare when attribution problems are present (Baliga et al., 2020), or when there is a hidden development of technological capabilities as the transgression (Meirowitz et al., 2008; Baliga and Sjöström, 2008; Schultz, 2010; Debs and Monteiro, 2014; Bas and Coe, 2016; Spaniel, 2019; Meirowitz et al., 2019). Second, while introducing the possibility of a continuum of transgression and hassling options constitutes a step in the direction of better describing international interactions, the transgression and hassling options examined here are both uni-dimensional. Building out a more sophisticated framework that allows for more dimensions of policy responses would even more accurately describe the world.

7 Conclusion

Here, we put forward a new class of models—flexible-response crisis bargaining models—and conduct a comprehensive analysis of them using the tools of mechanism design. These flexible-response crisis bargaining models represent a useful adaptation of the standard, dichotomous crisis bargaining framework, where war and peace are the only possible outcomes. In our modeling framework, states can engage in a continuum of conflict operations; this better captures the conditions of actual international crises, in which states select from an array
of possible conflict options (like gray zone conflict or sanctions). Rather than fix a single
game form and solve for a set of equilibria, in our analysis, we identify the properties shared
by all equilibria within the full class of flexible-response crisis bargaining games. This general
analysis allows us to be confident that our results are not driven by specifics of the game
form or a specific equilibria, but are generalizable to all flexible-response crisis bargaining
models.

Our most surprising results are those that differ from the Banks (1990) and Fey and Ramsay
(2011) monotonicity results. While existing research has shown that improved private war
capabilities or an improved private willingness to go to war can never decrease the likelihood
of war, we show that this relationship is more nuanced when war capabilities can also benefit
low-level conflict capabilities. Similarly, while Banks (1990) and Fey and Ramsay (2011) have
shown that an improved private ability to conduct war always produces a greater utility, we
find that these results do not necessarily hold when a robust ability to go to war can hurt
an actor’s ability to effectively sanction or hassle.

A central concern of international relations is understanding the drivers of costly and de-
structive conflict. While this topic has been well examined through models of war and peace,
much of what occurs in international relations falls outside of what could easily be classified
as a peaceful bargain or a decisive war. Naturally, any model must make some simplifying
assumptions on how the world works, but as this paper shows, neglecting the possibility for
low-level responses may lead to misunderstandings about what actually drives war. More
work is needed on this topic.

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