Conflicts that Leave Something to Chance: Establishing Brinkmanship Through Conventional Wars

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Abstract

I present a formal model of nuclear brinkmanship. The novelty here is that the threat of nuclear escalation comes about inadvertently, making it a function of conflict duration with a non-monotonic relationship to conventional capabilities. When two states have similar conventional capabilities and enter into a war, the conventional conflict will be prolonged, thus resulting in high degrees of nuclear risk. When two states have dissimilar conventional capabilities, conventional conflict will be one-sided and short, thus resulting in low degrees of nuclear risk. The model generates a series of results, including evidence of the nuclear peace, of conflict patterns consistent with the stability-instability paradox, and of the merits of "burning bridges." Additionally, the model generates mixed results on the feasibility of using strategic nuclear instability as a substitute for conventional arming.

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"Discussions of troop requirements and weaponry for NATO have been much concerned with the battlefield consequences of di erent troop strengths and nuclear doctrines. But the battlefield criterion is only one criterion, and when nuclear weapons are introduced it is secondary. The idea that European armament should be designed for resisting Soviet invasion, and is to be judged solely by its ability to contain an attack, is based on the notion that limited war is a tactical operation. It is not. What that notion overlooks is that a main consequence of limited war, and potentially a main purpose for engaging in it, is to raise the risk of larger war."

-Schelling, Thomas C. Arms and Influence

1 Introduction

The defining feature of international politics since 1945 has been the absence of direct great power conflict (Gaddis, 1986). To explain this historical anomaly, some neorealists classify this "long peace" as the "nuclear peace," where the fear of a nuclear exchange prevents significant conflict among great powers (Waltz, 1981; De Mesquita and Riker, 1982; Mearsheimer *et al.*, 2001; Waltz, 1981).[1] How the nuclear peace functions in practice is subtle. It is not as if states can credibly deter revisionist behavior through the threat of a nuclear first strike. After all, outside of circumstances where a state faces an existential threat, no state would ever launch a significant nuclear strike against a capable nuclear opponent, as doing so would be tantamount to suicide. And, the existence of nuclear weapons does not prevent states from fighting conventional wars. In theory, states with nuclear weapons could forgo these weapons and still fight conventional conflicts with one another, just as they did before the advent of nuclear weapons. Instead, what preserves the nuclear peace is the threat of indirect risk (Schelling, 1980, 1966; Powell, 2015). In the nuclear era, when states enter into a conventional conflict, that conventional conflict is unstable and could turn into a massive nuclear exchange. Proponents of the nuclear peace suggest that the near absence of large-scale, direct conflict between nuclear states stems from a new and frightening feature of conventional conflicts: they could accidentally spiral out of control (Snyder, 1965; Jervis, 2017; Schelling, 1966).

Schelling's insight (as expressed in the quote starting the paper) is that we must consider link-

[1]See Gaddis (1986) footnote 67.

ages between all aspects of conventional conflict—like troop placement and force posture—and nuclear risk. To begin unpacking this relationship, research has identified several mechanisms for how conventional conflicts could turn into nuclear exchanges. Accidents, decentralized decision making, or inadvertent escalation—all of which are discussed more below—could transform a conventional conflict into a nuclear one (Perrow, 2011; Sagan, 1994, 2020; Posen, 2014). Critically, across all these mechanisms linking conventional conflict to a nuclear exchange, there is a common underlying factor: time. When international conflicts between nuclear powers are short and decisive, there are fewer opportunities for an unintended escalation leading to a massive nuclear exchange. As conventional conflicts become protracted affairs, there are more times where these opportunities could arise. Adapting the Schelling (1966) metaphor, if a conventional conflict in the nuclear era is a war of nerves similar to "rocking the boat," then the shorter the time spent rocking, the less likely actors are to get soaked.

The positive relationship between conflict duration and nuclear risk presents a new dimension on what force posture means in the nuclear deterrence setting. Manipulating the conventional force posture can change how protracted a conflict is, which, in the nuclear era, changes the likelihood that a conventional conflict would turn nuclear. This was not an abstract notion. During the Cold War, deterring Soviet aggression against West Berlin did not depend on winning a conventional war; rather, deterrence required ensuring West Berlin had a sufficient conventional force posture to keep any conflict from ending decisively without any escalation risk. In contrast, in 1956, the Soviet Union committed a significant military force to quickly crush the Hungarian Revolution rather than risk letting the conflict spread and potentially escalate.

That additional arming could lead to more or less nuclear risk has fundamental implications for how troop placement can deter, yet this has not been formally considered. Overall, this means our understanding of how conventional arming, the placement of bases, and troop deployments are used in nuclear deterrence rests on shaky theoretical footing. I rectify this here.

I present a new model of nuclear deterrence through conventional arming. In the deterrence game, there are two actors, a defender and a challenger, where the defender's resolve is unknown to the challenger.[2] In the game, the defender begins by selecting a level of conventional arms.[3] Arming has several effects: (1) the defender incurs costs from arming, (2) the defender's likelihood of winning a conventional war is increasing in arming levels, and (3) should a conventional war occur, the likelihood of a nuclear war is non-monotonic in the level of conventional arming, depending on whether it will draw a conflict out or make a conflict more decisive. Next, the challenger observes this level of arming, and chooses whether to challenge or not. Finally, if challenged, the defender can acquiesce and back out of the challenge, or the defender can fight the challenger. When the defender fights the challenger, the actors fight a conventional war where there is a chance of a nuclear exchange.

The model produces a wide range of equilibria behavior, where conventional arming can serve as a bluff, a threat, a signal, or a means to perform well in a forthcoming war. Across all these potential behaviors, three sweeping results hold. First, consistent with the proposed "nuclear peace," as the possibility of a nuclear exchange resulting from conventional conflict becomes realized, actors will avoid engaging in conventional conflict. Second, consistent with the logic of the "stability-instability paradox," actors will primarily engage in fringe conflicts with limited nuclear stakes, and actors will arm with the intent of keeping nuclear conflicts less protracted. Finally, all equilibria exhibit a positive monotonic relationship between an actor's private resolve and their level of arming, even when this requires the defender to arm to a level beyond what they would have needed to deter a challenger.

The model also identifies new insights into arming in the nuclear era. In contrast to the logic of "asymmetric response" strategies, it is not possible for defenders use nuclear instability as a substitute for conventional arming in all cases. Because added nuclear instability dissuades both challengers ***and defenders*** from engaging in conventional conflict, in an environment with elevated nuclear instability, defenders must invest more in conventional armaments in order to be willing to fight when challenged. In other words, in a world with nuclear risk, a credible deterrent threat might require additional arming on the part of the defender. This also means that adding nuclear risk can have detrimental effects on welfare because it may require parties

[2]The structure here has similarities to many other deterrence models, including Fearon (1997), Powell (2015), Gurantz and Hirsch (2017), Baliga *et al.* (2020) Di Lonardo and Tyson (2022) as a few examples. See Huth (1999) and Ramsay (2017) for reviews.

[3]Manipulating p is analogous to deploying troops in preparation for a conflict.

to arm more to establish a credible deterrent threat; this insight, that nuclear risk can reduce welfare even when conflict does not occur, follows naturally from the model, and, to the best of the author's knowledge, is new.

This paper is most similar to the incomplete information model in Powell (2015) which also considers nuclear brinkmanship multiple levels of conflict. There are several differences, two of which I highlight here. First, these papers differ in how nuclear risk is generated. In this paper, nuclear risk is manipulated indirectly. The defender selects a level of arming that, among other effects, determines the expected conflict duration, which in turn determines the likelihood of a nuclear escalation. In Powell, the defender is able to directly manipulate nuclear risk, where such manipulation is public, credible, flexible, and will not alter the defender's likelihood of winning in a conventional conflict. Because these mechanisms for generating nuclear risk are different, deterrence functions differently across the models (as I discuss more in 7.6). Second, these papers differ in how conventional forces translate into nuclear risk. Powell also considers a challenger who selects a conventional force posture, where additional forces ***always*** results in greater nuclear risk. This paper is more flexible and assumes that adding conventional forces could generate more or less nuclear risk, depending on whether it makes the conflict more or less decisive.

Of course, this paper is also distinct from other signalling models with conflict. The perspective that arming or troop placement could reduce nuclear risk—when it leads to more decisive conflicts—or increase nuclear risk—when it drags out conflict—is different from other treatments in costly signalling games. Similar to work like Slantchev (2005) and Slantchev (2011), additional arming could lead to less nuclear risk and better expected wartime outcomes for the arming state, thus making it a productive costly signal. Or, additional arming could lead to more nuclear risk and worse expected wartime outcomes, thus making it a "handicap signal," as explored in Reich (2022). While existing research has explored these topics separately, there does not exist a unified, theoretical grounding for how conventional forces can function as a deterrent threat when arming could be productive or handicapping.

While this paper will present and discuss the model in the context of conventional war and nuclear brinkmanship, the formalization below can apply to other cases where actors engage in one level of conflict and there is the possibility of a costly and painful escalation. The "challenger" and "defender" below could be gangs or rival drug cartels that, when they engage in a protracted and bloody fight, run the risk of the government intervening. A similar logic could hold for low-levels of intrastate conflict that run the risk of a government or third-party actor intervention that is worse for both parties.

2 On Arming and Nuclear Risk

I consider interactions between a defender and a challenger. In the game, the defender initially fixes a level of conventional arming, and a challenger reacts to this. While much of the model below is fairly standard, what is different here is that conventional conflict could probabilistically escalate into a nuclear exchange. On this point, I assume a non-monotonic, increasing-then-decreasing relationship between conventional force levels and nuclear risk. I make this assumption based on three relationships, which relate conventional arming to nuclear risk. First, when the defender scales up their level of conventional forces, depending on the challenger's capabilities, this could result in greater conventional military parity between the challenger and defender or less military parity. Second, should a conventional conflict arise, a greater force conventional parity between disputants will result in a more protracted conventional conflict. Third, longer conventional conflicts generate a greater risk of a nuclear exchange. I elaborate on these below.

Adding conventional forces could result in more or less military parity. The first relationship is mechanical. Expanding conventional forces or building out military bases will either lead to greater military parity with a challenger—when the defender's capabilities approaches the challenger's capabilities—or less military parity with an opponent—when the defender's capabilities surpass the challenger's capabilities.

Greater military parity between actors results in longer conflicts. As intuition, if there is a low degree of military parity, then a quick, one-sided war or a rapid surrender is more plausible. On the other hand, if militaries are more evenly matched, then neither side has an immediate reason to stop fighting. Furthermore, because evenly-matched militaries will trade battle victories and defeats, war between equally matched adversaries will be less informative or less clearly decisive, which incentivises adversaries to continue fighting. This logic is illustrated in a series of theoretical models (Smith, 1998; Filson and Werner, 2002, 2004; Langlois and Langlois, 2009, 2012; Slantchev, 2004). Additionally, this does not only seem to be a theoretical finding, as the empirical literature does find evidence of a positive relationship between military parity and conflict duration. (Bennett and Stam, 1996, 2009; Slantchev, 2004; Krustev, 2006; Chiba and Johnson, 2019).[4]

Longer conflicts generate a greater likelihood of a nuclear exchange. Research has identified several mechanisms for how conventional conflicts could escalate to a nuclear exchange. This could come about entirely through accident. Within any complex system—like missile detection or early warning systems—system failures are possible (Sagan, 1994, 2020; Sagan and Waltz, 2003; Perrow, 2011). When states are at war, there is heightened risk that a faulty signal could be interpreted as an act requiring a nuclear response (Sagan, 1994). The possibility of a nuclear exchange could also come about through the course of conventional operations spiraling into a cycle of escalation. Whether through mechanical error (a malfunctioning GPS), human error (mis-reading maps), agency problems, or the fog of war, sometimes soldiers or operators take actions beyond what a fully rational, unitary decision maker would want, which could lead to a need for escalation (Sagan, 1994; Posen, 2014). Or, in a protracted conventional war, states may target their opponent's communication or command and control infrastructures, which could inadvertently undermine the targeted state's credible second-strike capability and necessitate an escalation (Posen, 2014).[5] Across all these different ways a conventional conflict could turn nuclear, time is a common underlying factor. When international conflicts between nuclear powers are short and decisive, there are fewer chances or reasons for system failures, overambitious operations, or the targeting of command and control infrastructure. As conventional conflicts drag on, the likelihood that these errors occur increases.

[4] This result does not always hold in a statistical sense. Bueno de Mesquita *et al.* (2004) finds this relationship within democratic dyads, but for non-democratic dyads, no statistically significant relationship exists. Similarly, Koch (2009) and Shannon *et al.* (2010) do not identify a statistically significant relationship. To the best of my knowledge, no existing research identifies a negative statistically significant relationship between military parity and conflict duration.

[5] Through the logic of mutually assured destruction, an actor on the verge of losing their second-strike capability might undertake dramatic, escalatory steps in an attempt to degrade their opponent's first-strike capability and thus preserve their second-strike capability.

To summarize the three relationships above, I will assume that the relationship between adding conventional forces to a conflict theater and nuclear risk is increasing or decreasing, depending on if it creates more or less military parity between the sides. This is a break with how Powell (2015) treats nuclear risk, where adding conventional forces to a conflict always results in greater nuclear risks within the conflict. Of course, for whatever reason, readers might take issue with my assumptions on this relationship; this is where the flexibility of this modeling effort is useful. While the model accommodates the increasing-then-decreasing relationship between conventional arming levels and nuclear risk, the results below still hold if the relationship between conventional arming and nuclear risk was ***only*** increasing or ***only*** decreasing.[6] This flexibility allows the model to describe a broader set of possible cases, making the results general to many different kinds of relationships between arming and nuclear risk.

3 Game Form and Assumptions

Two players, a challenger (C) and a defender (D), are in a deterrence game with incomplete information. The game order is as follows.

1. Nature designates D's resolve $v_D \in \{\underline{v}_D, \bar{v}_D\}$ with $0 < \underline{v}_D < \bar{v}_D$. Let π denote the probability that D is type $\bar{v}_D$.

2. D selects some conventional force level $p \in [p_0, p_1]$, with $0 < p_0 < p_1 < 1$.

3. C selects whether to challenge or not. If C does not challenge, the game ends with C receiving payoff 0 and D receiving payoff $v_D - K(p)$, where $K : \mathbb{R}_+ \to \mathbb{R}_+$ is D's costs from the conventional force level. I assume $K(p_0) = 0$, and K is continuous and increasing in p. If C does challenge, the game moves to the next stage.

4. D selects whether to acquiesce or escalate to conflict. If D acquiesces, C receives payoff v_C and D receives payoff $-K(p)$. If D escalates to conflict, then both states receive their conflict payoffs (described below).

[6]This can be accommodated by shifting the p_0 and p_1 parameters to only consider regions where the relationship is monotonic.

Conflict is a stochastic process that ends in one of three ways: C wins a conventional victory, D wins a conventional victory, or there is a catastrophic nuclear exchange. C's expected utility from conflict is

$$\frac{n}{h(p)} * (-N_C) + \frac{\alpha}{h(p)p(1-p)} ((1-p)v_C) - \frac{c_C}{h(p)},$$

and D's expected utility from conflict is [7]

$$\frac{n}{h(p)} * (-N_D) + \frac{\alpha}{h(p)p(1-p)} (pv_D) - \frac{c_D}{h(p)} - K(p).$$

Unpacking these terms, $-N_D$ and $-N_C$ denote D's and C's nuclear payoffs. p is the likelihood that D wins in a conventional war, conditional on the conflict process ending in a conventional war. The variable n is a "nuclear instability" parameter, where a greater n means, ***ceteris paribus***, a greater likelihood that a conventional war escalates to a nuclear war. Letting $h(p) = n + \frac{\alpha}{p(1-p)}$, the expression $\frac{n}{h(p)}$ denotes the likelihood that conflict ends in a nuclear exchange, and $\frac{\alpha}{h(p)p(1-p)}$ denotes the likelihood that conflict ends in a conventional victory or defeat. The expression $\frac{c_D}{h(p)}$ and $\frac{c_C}{h(p)}$ are the costs accrued from a conventional conflict.[8] I visualize the likelihood of a nuclear exchange and D's expected utility (without arming costs $K(p)$) from a conflict for a range of possible p's under one set of parameters in Figure 1.

First, in Figure 1, consider the likelihood of nuclear exchange, which is the solid line in the figure. For small or large conventional arming levels ($p \approx 0$ and $p \approx 1$), $h(p)$ becomes large and $h(p)p(1-p)$ becomes small. This means when the conventional arming level leads to a one-sided conventional war ($p \approx 0$ and $p \approx 1$), there is little risk of a nuclear exchange ($\frac{n}{h(p)}$ is smaller) and there is a greater likelihood of a conventional victory or defeat ($\frac{\alpha}{h(p)p(1-p)}$ is greater). In

[7] Or, for C and D (respectively), using $h(p) = \frac{\alpha + np(1-p)}{p(1-p)}$,

$$\frac{np(1-p)}{\alpha + np(1-p)} * (-N_C) + \frac{\alpha}{\alpha + np(1-p)} ((1-p)v_C) - \frac{c_C p(1-p)}{\alpha + np(1-p)}$$

$$-\frac{np(1-p)}{\alpha + np(1-p)} N_D + \frac{\alpha}{\alpha + np(1-p)} (pv_D) - \frac{c_D p(1-p)}{\alpha + np(1-p)} - K(p)$$

[8] As an alternate interpretation, n is the hazard rate of a nuclear exchange, $\frac{\alpha}{p(1-p)}$ is the hazard rate of conventional war termination, and costs accrue at constant rate c_D and c_C.

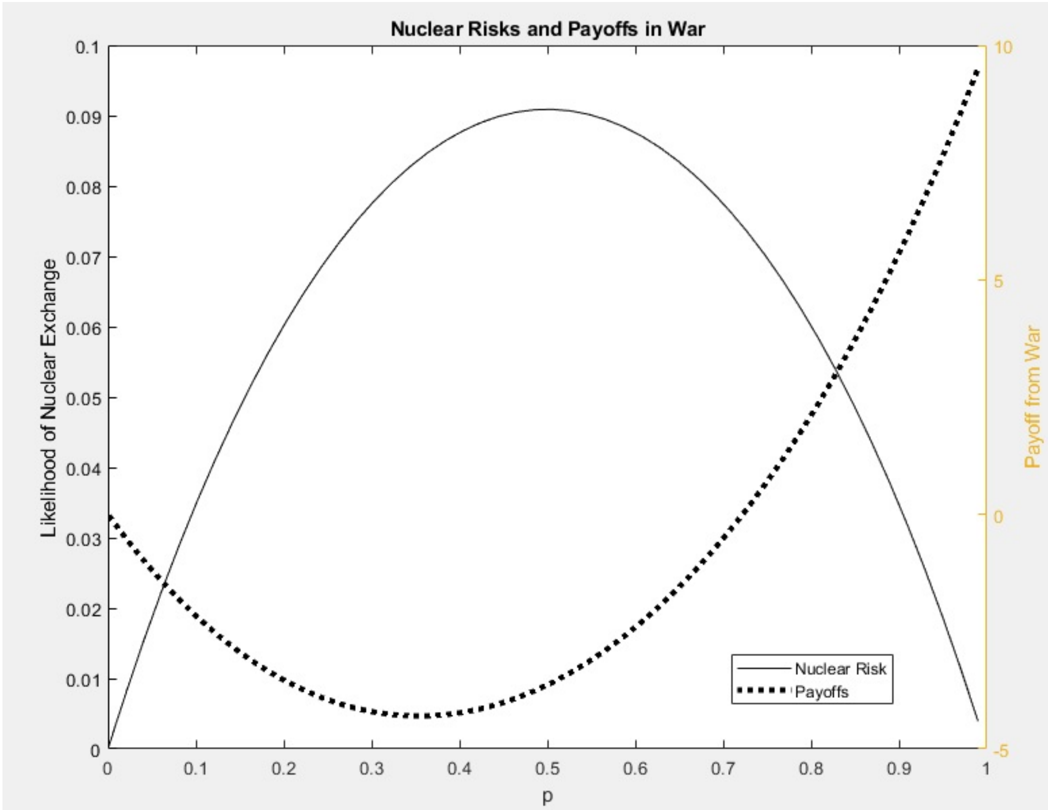

Figure 1: Nuclear risk and payoffs

contrast, in a more-balanced and protracted conventional war, there is greater risk of nuclear exchange and a (relatively) lower likelihood the game ends with a conventional victory or defeat. Now consider D's payoffs from conflict, which are captured in the dashed line in the figure. As p increases from 0 to roughly 0.4, the conflict becomes more protracted and the increasing risks of a nuclear exchange make D's utility decreasing. After this point, the defender's benefits from becoming more likely to win the conflict and the nuclear risk decreasing,[9] thus making D's utility increasing. Note that it could appear here that D's best option is to select the greatest possible arming level; recall that I am not plotting D's costs from arming $K(p)$, which would alter D's optimization problem.

4 Modeling Issues

This paper is most similar to the incomplete-information model in Powell (2015), but most clearly differs in the treatment of nuclear risk. In this paper, outside of exogenous parameters,

[9]The defender benefits in the $0.4 - 0.5$ range because the rate at which nuclear risk increases is decreasing.

nuclear risk is fully determined through the defender's arming level.[10] In the Powell formalization, after the challenger selects a conventional arming level, the defender is able to publicly and credibly manipulate the level of nuclear risk within a conventional war without altering their likelihood of winning in the conventional war.[11] A natural interpretation of the defender's choice in the Powell model would be manipulating Defense Readiness Condition (DEFCON) levels while in a conflict, which, through organizational and technical channels, would alter levels of nuclear risk (Sagan, 1985).

While Powell is groundbreaking in what it does, the model there may not apply to all settings. First, Powell assumes it is possible to publicly and credibly manipulate nuclear risk within a crisis. However, practically, doing so is subject to "cheap-talk" concerns. In Powell, in many cases, a defender would want to signal that they have implemented a high-risk (of nuclear exchange) system when in fact they have implemented a low-risk system. As a second limitation, Powell assumes it is possible to exclusively manipulate nuclear risk without altering the balance of conventional power. This feature means that Powell's model is outside of most standard discussions of tripwires (see Schelling (1966)); for example, the positioning of conventional forces in Western Berlin could be used to generate nuclear risk, but it also would mechanically alter the likelihood of conventional conflict success.[12] Third, in the Powell model, taking the defender's choice as fixed, there is a positive monotonic relationship between conventional force levels the challenger brings and the final probability of a nuclear exchange. In this regard, Powell is not considering the possibility that arming to a level that produces a decisive victory could ever reduce the probability of a nuclear exchange.[13]

Additionally, because nuclear risk is generated differently across the models here and in Powell (2015), signalling resolve and deterrence function differently. This is discussed in more length

[10] This is represented in the $\frac{n}{h(p)}$ function.

[11] Formally, after a challenger selects some level of arming p, the defender selects function $r(p)$ from a set of allowed functions. p denotes the likelihood the C wins in a conventional conflict, and $r(p)$ denotes the likelihood the conventional conflict escalates to a nuclear exchange.

[12] Powell (2015) discusses the Kremlin's decision whether or not to authorize the use of tactical nuclear weapons in Cuba as an example of manipulating $r(\cdot)$. However, using tactical nuclear weapons would undoubtedly alter the defender's likelihood of winning a conflict, which means that p would also be altered. Additionally, the Kremlin's decision was not known to the United States, meaning it was not publicly manipulated.

[13] I would be remiss to highlight two things Powell does quite well. First, while the Powell risk function is only monotonically increasing in the conventional arming level, this risk function is not assigned any functional form (like it is here) and is allowed to be quite flexible. Second, Powell considers an endogenously arming challenger while I treat the challenger's arming level as exogenous.

in Section 7.6.

In addition to (Powell, 2015), this paper also has benefited from decades of iterations of models of nuclear deterrence (Schelling, 1980; Nalebuff, 1988; Zagare and Kilgour, 1993; Powell, 1989, 2003). Because this is not a review article I will not cite the entire set of this research on nuclear deterrence, but rather refer readers to several excellent reviews, including see Jervis (1979), (Huth, 1999), Quackenbush (2011) and Gartzke and Kroenig (2016).Additionally, the model integrates features from the formal literature on endogenous power shifts and deterrence (Fearon, 1997; Schultz, 2010; Tarar, 2016; Debs and Monteiro, 2014; Gurantz and Hirsch, 2017).

Of course, nearly all models cited above only consider two types of outcomes: war or peace. A new branch of research relaxes assumes that conflict can be more multifaceted and take on more forms. (Zagare and Kilgour, 1998; Werner, 2000; Schultz, 2010; Slantchev, 2011; Powell, 2015; Tarar, 2016; McCormack and Pascoe, 2017; Coe, 2018; Spaniel, 2019; Joseph, 2020; Baliga *et al.*, 2020; Schram, 2021a,b). The key distinctions from existing work is that (a) I consider uncertainty over the defenders valuation of the good, and (b) to better capture the case of nuclear escalation, I consider a stochastic structure to when conflict escalates from lower-levels to higher-levels. To the best of my knowledge, no existing paper does both.

5 Complete Information Equilibrium

Before discussing the incomplete-information game, I first establish the equilibrium for a complete-information version of the game. This version of the game no longer has nature setting D's resolve as $v_D \in \{\underline{v}_D, \bar{v}_D\}$, but rather defines D's resolve as fixed v_D.

The key strategic tension in this game is as follows. D is only willing to fight if they have sufficiently armed; for D to fight when challenged, D must first select a level of arming at or beyond a cutpoint that I will refer to as p^D. Essentially, for any level of arming (weakly) greater than p^D, D does well enough in a conventional conflict to be willing to incur the costs of the conventional conflict and the probability of escalation. And, C will challenge unless two conditions hold: (a) C knows that they will have to fight after challenging, and (b) C knows that fighting is sufficiently bad for them. For (b), if D's level of arming is at or beyond some cutpoint (that I will refer to as p^C), then C will do sufficiently bad should a fight arise. For (a), to keep C from challenging, it must be that D is willing to fight when challenged, meaning that D has armed at least to level p^D.

I first define p^C as the conventional force level that would make C indifferent between challenging or not, conditional on D escalating in stage 4.[14]

$$p^C = \frac{\alpha v_C}{c_C + nN_C}$$

I also define p^D as the conventional force level that would make D willing to escalate conditional on C challenging.[15]

$$p^D = 1 - \frac{\alpha v_D}{c_D + nN_D}$$

For ease, I will assume for the complete information case that $p^C > p_0$, $p^D > p_0$, $p^C < p_1$, $p^D < p_1$. Together, these expressions imply that deterrence is feasible when the costs are low enough.

Also note that if $p^D < p^C$, D could select some $p \in [p^D, p^C]$, representing the optimal conventional force level knowing that C will challenge, D will escalate, and the game will end in war.[16] I define

$$\hat{p} \in argmax_{p \in [p^D, p^C]} \left\{ \frac{n}{h(p)} * (-N_D) + \frac{\alpha}{h(p)p(1-p)} (pv_D) - \frac{c}{h(p)} - K(p) \right\},$$

where $\hat{p}$ is the optimal conventional force level. Note that it is possible that the set $\hat{p}$ is not singleton, in which case I abuse notation and let $\hat{p}$ define the smallest selected arming level. I define $U_D(\hat{p})$ as D's utility from selecting $\hat{p}$ as defined above.

In equilibrium, D will select one of three arming values. First, D could select $p^* = max \left\{ p^C, p^D \right\}$ which deters C from ever challenging and gives D a final utility of $V_D - K(p)$. Second, D could select $p = p_0$ and acquiesce when challenged, giving D a final utility of 0. Finally, it could be

[14] Formally, $0 = -\frac{n}{h(p^C)} N_C + \frac{\alpha}{h(p^C)p^C(1-p^C)} \left((1-p^C)v_C \right) - \frac{c}{h(p^C)}$.

[15] $0 \leq \frac{n}{h(p^D)} * (-N_D) + \frac{\alpha}{h(p^D)p^D(1-p^D)} \left(p^D v_D \right) - \frac{c_D}{h(p^D)}$

[16] As is common in deterrence models, war is possible in this complete information game when states both value the asset highly enough.

that deterring is too costly but fighting is more productive than peace: in this case, D will select some $p = \hat{p}$. Formally, the equilibrium play is as follows.

Case 1: Let $p^C \leq p^D$.

1A. If $V_D - k(p^D) \geq 0$, then D selects $p = p^D$ and C does not challenge.

1B. Otherwise, D selects $p = p_0$, C challenges, and D acquiesces.

Case 2: Let $p^D < p^C$.

2A. If $V_D - k(p^C) \geq 0$ and $V_D - k(p^C) \geq U_D(\hat{p})$, then D selects $p = p^C$ and C does not challenge.

2B. If $0 > V_D - k(p^C)$ and $0 > U_D(\hat{p})$, then D selects $p = p_0$, C challenges, and D acquiesces.

2C. Otherwise, D selects $p = \hat{p}$, C challenges, and D fights.

While the primary analysis occurs below, note that p^D is decreasing in v_D. This implies that as D has a greater resolve, D can deter through lower levels of arming in the complete information model. This is important because in the incomplete information version of the game, moving from type $\underline{v}_D$ to type $\bar{v}_D$ ***always*** results in a greater level of arming. As I will discuss, this means that we should expect arming to be inefficient in the game with incomplete information.

6 Incomplete Information Equilibria

In the incomplete information game, much of the strategic tension is similar to what is described in the complete information game. What's new here is that because C is uncertain of D's resolve, sometimes C is uncertain whether D is willing to fight or not. This in turn can shape arming decisions. In many equilibria, low-types and high-types will behave differently. For example, sometimes low-types will acquiesce, and high-types will deter. More interesting behavior can also arise. Under some parameters, low-type D's can bluff by mimicking the arming behavior of high-type D's; this can result in C not challenging low-type D's even when low-type D's would not fight if challenged. Also, under different parameters, sometimes high-type D's will signal their resolve by selecting levels of arming beyond what was needed in the complete-information game, with the intent of getting low-type D's to drop out.

I assign some additional structure to the analysis. Because this is an incomplete information game, there are many possible Nash equilibria. For that reason, I refine the set of possible equilibria.

Equilibrium Assumptions: I only consider perfect Bayesian Nash equilibria that satisfy the intuitive criterion (Cho and Kreps, 1987).

While multiple equilibria can still exist after applying the intuitive criterion—for example, when C or D is indifferent over their actions a continuum of strategies could still be supported—the empirical implications of the remaining multiple equilibria will be limited.[17] For the rest of this paper, I focus on a single equilibrium and conduct analysis of it.

Because this model takes a phenomenonalist approach (Paine and Tyson, 2020) where the model's setup corresponds with trade-offs that decision-makers face, I make few restrictive assumptions and the model can generate a large set of possible strategic behavior (deterring, bluffing, signaling, etc). For that reason, I would encourage readers to not get too bogged-down in every equilibrium statement in this section, especially because the next sections discussing general results across all equilibria are the better "punchlines" that should be considered. This being said, because it is not difficult to find real-world examples of signaling, bluffing, deterring, or acquiescing within international relations, efforts made to further simplify the set of possible equilibria would detract from the generality of the analysis.

6.1 Characterizing Equilibrium Arming Levels

Before presenting the equilibria, I define several of the arming levels that are selected in equilibria, and I offer some intuition for their relevance. I derive all values in the Appendix.

As it was above, I define p^C as the conventional force level that would make C indifferent between challenging or not, conditional on D escalating in stage 4.

$$p^C = \frac{\alpha v_C}{c_C + nN_C}$$

[17] For example, all equilibrium actions (i.e. the selected p^*, the decision to challenge, and the decision to fight) listed below could be supported by a range of possible off-path beliefs. Alternatively, multiple equilibria can arise when C or D has indifference over their set of actions. For example, under some parameters, high-type D's will be indifferent between selecting an arming level where they will drop out when challenged or arming up with the intent of fighting when they are challenged. This indifference (or others like it) can support a continuum of mixed strategy equilibria that I am not considering.

I let $p^D(v_D)$ denote the conventional force level that would make a type $v_D \in \{\underline{v}_D, \bar{v}_D\}$ D indifferent between escalating or not conditional on C challenging.

$$p^D(v_D) = 1 - \frac{\alpha v_D}{c_D + nN_D}$$

Additionally, I define $\tilde{p}$ as the smallest feasible level of arming that would both (a) make type $\bar{v}_D$ D escalate when challenged and type $\underline{v}_D$ D acquiesce when challenged, and (b) make C at-least indifferent between challenging or not conditional on D's expected behavior. As intuition, if a $\tilde{p}$ exists, then there can exist a pooling equilibrium where both types of D select $\tilde{p}$ and C never challenges. To that end, because for arming level $\tilde{p}$ type $\underline{v_D}$ D's will not fight, here $\underline{v_D}$ D is bluffing. To define $\tilde{p}$, I first characterize the set $\tilde{P}$ as all values of p that satisfy conditions (b) and (a) in the order listed.

$$\tilde{P} = \left\{ p : \begin{cases} 0 \geq & \pi \left(-\frac{n}{h(\tilde{p})} N_C + \frac{\alpha}{h(\tilde{p})\tilde{p}(1-\tilde{p})} \left((1-\tilde{p}) v_C \right) - \frac{c_C}{h(\tilde{p})} \right) + (1-\pi) v_C, \ and \\ p \in & \left[max \left\{ p^D(\bar{v}_D), p_0 \right\}, min \left\{ p^D(\underline{v}_D), p_1 \right\} \right] \end{cases} \right\}$$

Second, I define $\tilde{p}$ as $\tilde{p} = min \left\{ \tilde{P} \right\}$ whenever $\tilde{P}$ is non-empty.

Next, I define $\hat{p}(v_D)$ as the level of arming level that is optimal for a type v_D D conditional on C challenging and D fighting.[18] I let $\hat{U}_D(p, v_D)$ denote a type v_D D's utility from selecting arming level p, C challenging, and D fighting.[19]

$$\hat{p}(v_D) \in \underset{p \in [max\{p^D(v_D), p_0\}, min\{p^C, p_1\}]}{\arg\max} \left\{ \hat{U}_D(p, v_D) \right\}$$

Note that the set $\left\{ p : p \in \left[max \left\{ p^D(v_D), p_0 \right\}, min \left\{ p^C, p_1 \right\} \right] \right\}$ may be empty, meaning no such $\hat{p}(v_D)$ exists.

Finally, I implicitly define $\bar{p}$ as the level of arming where a low-type D is indifferent between (a) arming to level $\bar{p}$, not being challenged, and attaining the asset, and (b) arming to level p_0,

[18] Formally, for both C and D to be willing to fight, it must be that D selects a p such that p is greater than $p^D(\bar{v}_D)$ and less than p^C, while also subject to the bounds that $p \in [p_0, p_1]$.

[19] Formally, $\hat{U}_D(p, v_D) = \frac{n}{h(p)} * (-N_D) + \frac{\alpha}{h(p)p(1-p)} (pv_D) - \frac{c_D}{h(p)} - K(p)$.

Symbol	Value
p^C	$p^C = \frac{\alpha v_C}{c_C + nN_C}$
$p^D(\bar{v}_D)$	$p^D(\bar{v}_D) = 1 - \frac{\alpha \bar{v}_D}{c_D + nN_D}$
$p^D(\underline{v}_D)$	$p^D(\underline{v}_D) = 1 - \frac{\alpha \underline{v}_D}{c_D + nN_D}$
$\tilde{p}$	$0 \geq \pi \left(-\frac{n}{h(\tilde{p})} N_C + \frac{\alpha}{h(\tilde{p})\tilde{p}(1-\tilde{p})} \left((1-\tilde{p})v_C\right) - \frac{c_C}{h(\tilde{p})} \right) + (1-\pi)v_C$
$\hat{p}(v_D)$	$\hat{p}(v_d) \in \underset{p \in [max\{p^D(\bar{v}_D), p_0\}, min\{p^C, p_1\}]}{\arg\max} \left\{ \frac{(-n*N_D - c_D)}{h(p)} + \frac{\alpha}{h(p)p(1-p)} (pv_D) - K(p) \right\}$
$\bar{p}$	$\underline{v}_D - k(\bar{p}) = 0$

Table 1: D's selected arming levels.

always being challenged, and acquiescing.

$$\underline{v}_D - K(\bar{p}) = 0.$$

I summarize the arming levels in Table 1.

For simplicity, I make the following parameter assumptions.

Parameter Assumptions: I assume $\bar{v}_D - K(max\left\{p^D(\bar{v}_D), p_0\right\}) > 0$ and $p^1 > p^D(\underline{v}_D)$.

The Parameter Assumptions imply that type $\bar{v}_D$ D's are able to fight over the asset (that arming level $p^D(\bar{v}_D)$ can be reached because $p^D(\bar{v}_D) < p_1$),[20] and that they are willing to arm to this level if it means that they attain the asset ($\bar{v}_D - K(max\left\{p^D(\bar{v}_D), p_0\right\}) > 0$). Essentially, if these assumptions did not hold, then high type D's would never be willing to deter C. I allow more flexibility for low-types. I also assume that type $\underline{v}_D$ D's are able to deter (that arming level $p^D(\underline{v}_D) < p_1$), but I do not assume that low-type D's are always willing to deter (I do not assume that $\underline{v}_D - K(max\left\{p^D(\underline{v}_D), p_0\right\}) > 0$).

6.2 Discussion

Table 2 presents the selected arming levels and whether or not war occurs. See the Appendix for a complete characterization. For the values in Table 2, I assume $p_0 < p^D(\bar{v}_D)$; I express the full equilibria without this assumption in the Appendix.

Within Table 2, I characterize eight equilibria based on the strategic play of high- and low-

[20] Note that if $p^1 > p^D(\underline{v}_D)$ holds, $p^1 > p^D(\bar{v}_D)$ also holds.

Equilibrium Behavior	**Equilibrium Type**	**Type $\bar{v}_D$ arming**	**Type $\underline{v}_D$ arming**
Acquiesce-Acquiesce	Pooling	p_0	p_0
Deter-Bluff	Pooling	$max\left\{p^D(\bar{v}_D), \tilde{p}\right\}$	$max\left\{p^D(\bar{v}_D), \tilde{p}\right\}$
Deter-Deter	Pooling	$max\left\{p^D(\underline{v}_D), p^C\right\}$	$max\left\{p^D(\underline{v}_D), p^C\right\}$
Signal-Acquiesce	Separating	$\bar{p}$	p_0
Deter-Acquiesce	Separating	$max\left\{p^D(\bar{v}_D), p^C\right\}$	p_0
Fight-Acquiesce	Separating	$\hat{p}(\bar{v}_D)$	p_0
Fight-Fight	Separating	$\hat{p}(\bar{v}_D)$	$p^D(\underline{v}_D)$
Deter-Fight	Separating	p^C	$p^D(\underline{v}_D)$

Table 2: Equilibrium Summary. Note the values here assume $p_0 < p^D(\bar{v}_D)$.

types, as described in the first column. The second column denotes whether the equilibrium behavior is part of a pooling or separating equilibrium. The third and forth column denote the high-type's and low-type's arming levels.

Regarding equilibrium behavior, either type D can "acquiesce," meaning they select the minimum arming level $p^* = p_0$ and acquiesce when challenged. Either type D can "deter," or select a level of arming that would keep C from challenging and where they would fight if challenged. To deter, high-types must select at least $p^* \geq max\left\{p^D(\bar{v}_D), p^C\right\}$, and low-types must select at least $p^* \geq max\left\{p^D(\underline{v}_D), p^C\right\}$. Either type D can "fight," or select a level of arming knowing that C will challenge and they will escalate and fight. Here D will select their optimal arming level $p^* = \hat{p}(v_D)$. Only low-types "bluff." Bluffing describes behavior within a pooling equilibrium where low-types select a level of arming where they (a) mimic high-type's arming level and (b) would not fight if challenged ($p^* < p^D(\underline{v}_D)$). In the bluffing equilibrium space, both types of D select $\tilde{p}$. Only high-type players "signal." When high-types signal, high-types select a level of arming beyond what they would need to deter in a complete information game as a way distinguish themselves from low-type D's. In the signalling equilibrium space, high types will select $p^* = \bar{p}$, where $\bar{p}$ is the arming level that makes low-type D indifferent between arming and acquiescing; thus, in the separating equilibrium space, low-types will drop out and acquiesce.

Figure 2 illustrates one example of equilibrium behavior. On the x-axis, I vary $\underline{v}_D$ (increasing left-to-right). On the y-axis, I vary v_C (increasing low-to-high). In the region where low-type D's have the lowest resolve and v_C is low (bottom left), high-type D's can arm to a level that will keep C from challenging, and low-type D's are unwilling to arm to this level: this is the

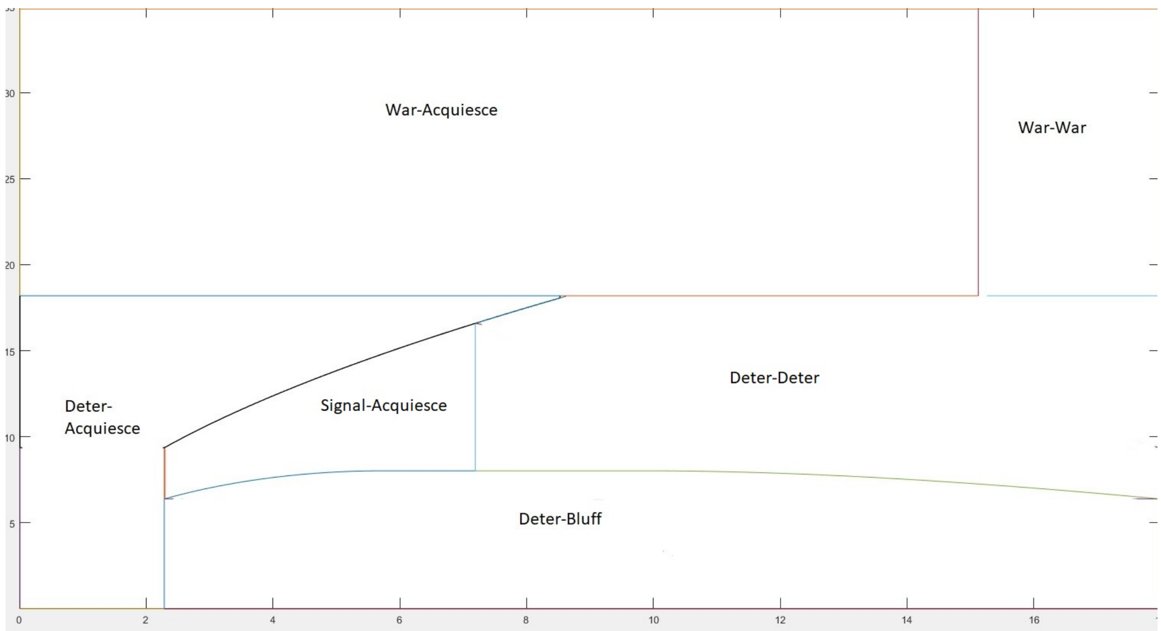

Figure 2: Equilibrium behavior. The x-axis varies $\underline{v}_D$ and the y-axis varies v_C.

"Deter-Acquiesce" equilibrium space.[21] As v_C grows (moving up), it either becomes too costly for high-types to deter C ($K(p^C)$ is too high) or it no longer becomes feasible to deter C ($p^C > p_1$). When this is the case, high-type D's will switch from deterring either to going to war or to acquiescing. Under the parameters used here, the high-type D's prefer going to war over acquiescing. Note that moving to the right of the War-Acquiesce space—increasing $\underline{v}_D$ to a point that approaches $\bar{v}_D$— moves into the War-War equilibrium space, where both types select an arming level that will result in C challenging, and both types fighting. In the upper right corner, because low-types look more like high types (i.e. they care almost as much about the issue), they are also willing to fight.

In the "Deter-Bluff" equilibrium space, low-type D's care more about the issue because $\underline{v}_D$ has increased, but C does not. In this space, both D's are willing to select some arming level where high-type D's would be willing to fight if challenged ($p^* \geq p^D(\bar{v}_D)$), where low-type D's would be unwilling to fight if challenged ($p^* \leq p^D(\underline{v}_D)$), and where C, knowing this, isn't resolved enough to challenge. Here low-type D's can successfully bluff without the bluff ever being called. As C cares more about the issue (moving up), one of two things can happen. When low-type

[21]This space has the curve above the Signal-Acquiesce equilibrium space because at the start of the curve $p^C = p^D(\bar{v}_D)$. Then, as v_C increases, p^C increases while $p^D(\bar{v}_D)$ stays the same, making deterrence more costly; thus, under more values of $\underline{v}_D$, low-type D's are unwilling to fight.

D's could have fairly low resolve (i.e. the Signal-Acquiesce region), high-type D's can signal to distinguish themselves from low-type D's by selecting an arming level $p^* = \bar{p}$ that low-type D's are not willing to arm up to ($\underline{v}_D - K(\bar{p}) \leq 0$). This costly signal allows C to know that only high types would select $p^* = \bar{p}$, and this will motivate C to not challenge when observing $\bar{p}$ because then C would always have to fight. When low-type D's care more about the issue (moving to the right), this is the "Deter-Deter" equilibrium space where both types of D's are willing to arm up to level p^* such that low-type D's would fight (i.e. $p^* \geq p^D(\underline{v}_D)$) and where C would be deterred from challenging $p^* \geq p^C$.

Two equilibrium spaces—"Acquiesce-Acquiesce" and "Deter-War"— are listed in Table 2 but not graphed. These are fairly straitforward. Under different assumed parameters (or functional forms), these are possible, and would occur in the upper-regions of the graph.

7 Results

All proofs are in the appendix.

7.1 Arming Levels are Increasing in Private Type

Remark 1 (Monotonicity in Arming). *High types always select greater arming levels ($p^*(\bar{v}_D) \geq p^*(\underline{v}_D)$).*

In order for D to be willing to fight when challenged, D must arm to at least level $p^D(v_D) = 1 - \frac{\alpha v_D}{c_D + nN_D}$. Were this a complete information game and both low- and high-type D's were trying to deter C from challenging, high-type D's could deter C with lower levels of arming than low-type D's (because $p^D(v_D)$ is decreasing in v_D).

But, in the incomplete information game, arming is ***always*** increasing in private type. Why? Some of the logic for Remark 1 can be illustrated within the Deter-Deter equilibrium space when both types select $p^D(\underline{v}_D)$, which is beyond what high-types need to be willing to fight. Within this equilibrium space, high-types deviate cannot deviate and select lower levels of arming due to the standard costly-signalling logic (Cho and Kreps, 1987). Suppose all high-types switched and selected $p' = p^D(\bar{v}_D)$ (with $p^D(\bar{v}_D) > p^C$); if this were the case, then C would not challenge, as C would know that the high-types who selected arming level p' would fight if challenged. However, this cannot be an equilibrium because low-type D's would have an incentive to mimic high-types by selecting arming level p' (which is less costly) and not get challenged.

There are two subtleties to Remark 1 worth mentioning. First, recall that in some cases D will arm with the intent of deterring C, and in other cases D will arm with the intent of fighting D in a conventional war. Because deterring C requires a greater arming level than fighting with C,[22] Remark 1 implies that equilibria can exist where low-type D's fight and high-type D's do not. This is consistent with other models of deterrence like Slantchev (2005). Figure 3 illustrates one example of this.

The top graph in Figure 3 illustrates low-type D's equilibrium arming levels (p, on the y-axis) over a range of C's valuation of the asset (v_C, on the x-axis). The bottom graph illustrates high-type D's equilibrium arming levels over the same range of C's valuations. The key take-away from this figure is that, under this parameterization, there is a range of v_C's where low-type D's fight but high-type D's do not (roughly 30.85 to 31.25). This equilibrium behavior is consistent with Remark 1, as every low-type D selects a weakly lower level of arming for a given v_C. To elaborate on what is happening, consider what would happen if D was facing a Challenger with valuation $v_C = 31$. A low-type D has three arming options: (1) they could select some optimal arming level in preparation for fighting ($p = 0.65$); (2) they could arm to deter C and attain the asset with certainty ($p = 0.85$); or (3) they could not arm and acquiesce when challenged ($p = p_0$). For these parameters and all possible v_C, the not-arming option (3) is always worse than fighting (1) or deterring (2). Additionally, at $v_C = 31$, deterring C is too costly for low-type C's, and low types prefer arming to a lower-level and fighting relative to maintaining the high level of arming needed to actually deter C. In contrast, when $v_C = 31$, high-type D's prefer deterring to fighting . Because high-type D's value the asset more than low-types, entering into a "costly lottery" over the asset is less appealing for high types than getting the asset outright; thus high types are willing to arm to high levels to deter. Of course, even high-type D's have their limits. At $v_C = 31.25$, the costs of arming are too high even for high-type D's, and high types switch and prefer to fight.

[22]Fighting is only possible when $p^D(v_D) < p^C$ and D selects some $p \in [p^D(v_D), p^C)$; meanwhile, when feasible, D can deter C by setting $p = p^C$.

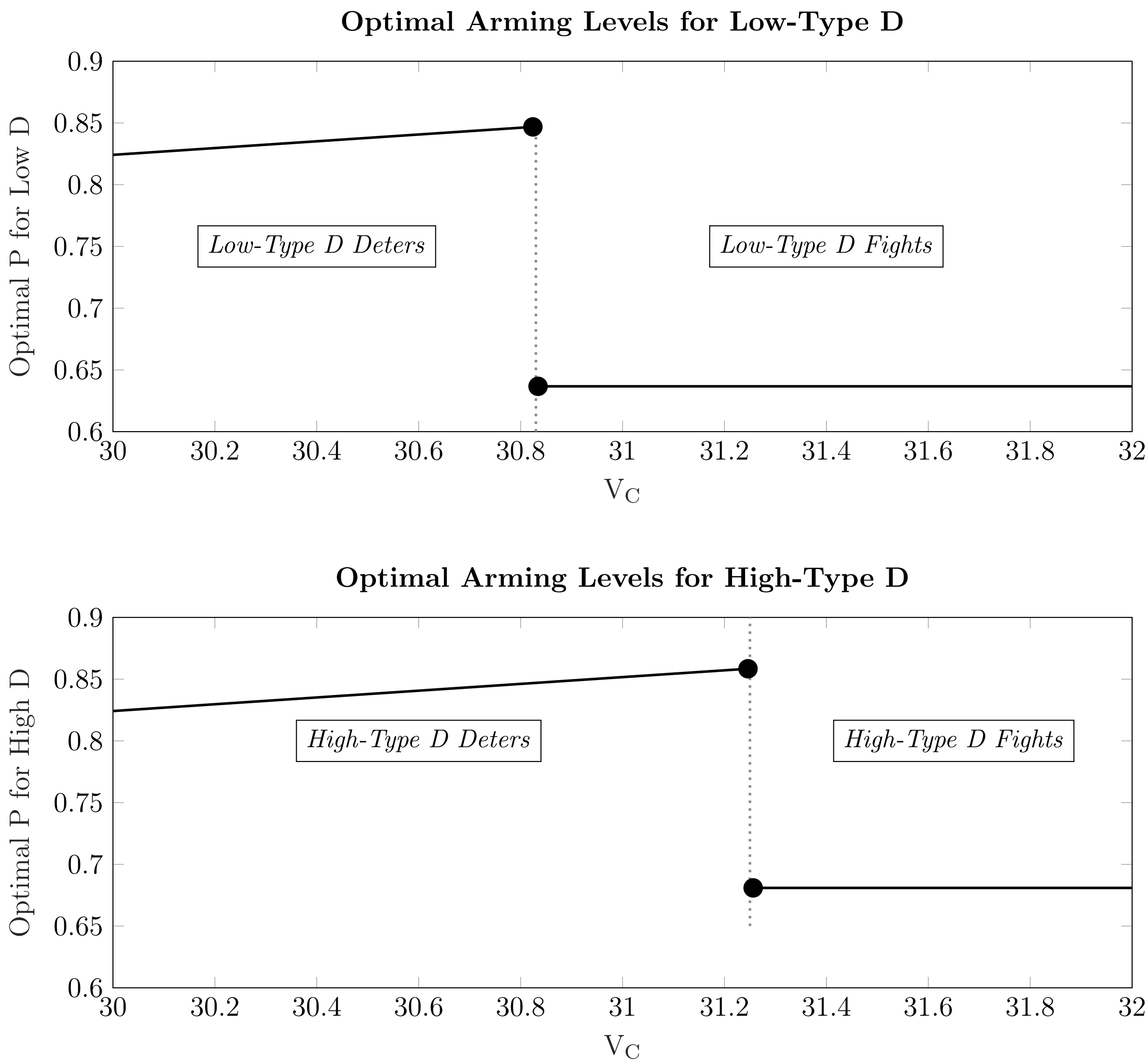

Figure 3: Optimal Arming Levels for D at Different Values of C's Resolve

The x-axis varies the values of $\underline{v}_C$, and the y-axis displays D's optimal arming levels for low-types (top graph) and high-types (bottom graph). The labels indicate where D deters or where D fights.In addition to confirming Remark 1 (the optimal level of arming is weakly increasing in private type), the key take-away is that under some parameters, low-types fight while high-types do not. The cost function is $k * (p^* - p_0)/(p_1 - p^*)$. The non-illustrated parameter values are $p_0 = 0.001$, $p_1 = 0.99$, $c_D = 0.8$, $c_C = 1.5$, $N_C = 32$, $N_D = 40$, $\alpha = 0.5$, $n = 0.01$, $k = 3$, $\pi = 0.8$, $\underline{v}_D = 18$, and $\overline{v}_D = 23$. (DRAFT NOTE): This figure is incomplete: there should be an open dot, and the dotted line should go to the bottom.

As the second subtlety, Remark 1 rules out a range of possible equilibria behavior. Recall that D "acquiescing" involves the lowest level of arming (p_0), fighting involves a greater level of arming ($\hat{p}(v_D)$), and deterring involves the greatest level of arming ($max \left\{p^D(v_D), p^C\right\}$. Remark 1 implies that it will never be the case that high-type D's acquiesce when low-type D's deter or fight, and high-type D's will never fight when low-type D's deter.

7.2 Introducing Nuclear Risk Requires More or Less Conventional Arming for Deterrence

Remark 2 (Nuclear and Conventional Forces are Imperfect Substitutes). *Increasing nuclear instability n has ambiguous e ects on the equilibrium level of conventional arming (p).*

Increasing the level of nuclear instability could result in D optimally selecting greater or lower levels of conventional arming. This ambiguity is cleanly seen when D is trying to deter C (for example, in the Deter-Acquiesce equilibrium). Within this equilibrium space, $p^* = max \left\{p^D(\bar{v}_D), p^C\right\}$, meaning that to deter C, D must arm to a level that will make D willing to escalate and will keep C from challenging. Consider the case when the relevant constraint is D's own willingness to fight ($p^D(\bar{v}_D) \geq p^C$), meaning D selects arming level $p^* = p^D(\bar{v}_D)$. Based on how $p^D(\bar{v}_D)$ is defined, $p^D(\bar{v}_D)$ is increasing in n. Intuitively, as nuclear instability increases, for D to be willing to escalate, D must attain a better outcome in the conventional conflict to compensate for the greater nuclear risk from fighting. Next, consider when the relevant constraint is C's willingness to fight ($p^D(\bar{v}_D) < p^C$), meaning D selects arming level $p^* = p^C$. By how p^C is defined, p^C is decreasing in n. Intuitively, as nuclear instability increases within a conventional war, C becomes less willing to challenge and end up in this conflict. Thus, depending on underlying model parameters, the effect of increasing nuclear instability on D's selected arming level is ambiguous.

Remark 2 captures the difficulties in attempting to use nuclear risk as a substitute for conventional capabilities. Waltz (1981) describes several reasons why states may want nuclear weapons, including the following: "[S]ome countries may find nuclear weapons a cheaper and safer alternative to running economically ruinous and militarily dangerous conventional arms races. Nuclear weapons may promise increased security and independence at an affordable price.” Within the scope of this paper—where a strategic nuclear exchange is a background risk within a conventional war—only sometimes can nuclear weapons serve as an alternate to conventional arming.[23] In some cases, increasing nuclear risk makes D less willing to fight over issues, which means that unless D invests in a robust conventional capability to ensure a conventional war with C goes more in their favor, D will not be willing to fight when challenged.[24]

Result 2 also has clear policy implications. Consider the 2010 Nuclear Posture Review, which states that “fundamental changes in the international security environment in recent years—including the growth of unrivaled US conventional military capabilities [and] major improvements in missile defenses...enable us to fulfill...objectives at significantly lower nuclear force levels and with reduced reliance on nuclear weapons...without jeopardizing our traditional deterrence and reassurance goals” (Leah and Lowther, 2017). On one hand, the model partially supports this assessment, as a robust conventional force posture can sometimes deter in settings where nuclear instability is lowered. On the other hand, we have not observed the counterfactual world where nuclear instability has been lowered; if this instability was pivotal in deterring rivals, reducing reliance on nuclear weapons could require an expansion in conventional forces. Thus, it remains an open question how the scope of challenges would change when nuclear instability is lowered.

7.3 Evidence of a “Nuclear Peace”

Remark 3 (Nuclear Peace). *Consider nuclear instability parameters $n', n'' \in \mathbb{R}_+$ with $n' < n''$. For both $v_D \in \{\underline{v}_D, \bar{v}_D\}$, so long that either*

(a) For n'', $p^C \leq p^D(\bar{v}_D)$ holds, or

(b) For n'', $p^C > p^D(\bar{v}_D)$ holds, and for n' and n'' $\hat{U}_D(p, v_D)$ is concave in p,

then the shift from n' to n'' results in less war.

Remark 3 implies that moving from a lower to higher nuclear instability parameter will shrink the parameter set under which a conventional war will occur. This has empirical implications. One way to formalize the introduction of nuclear weapons is increasing the nuclear instability

[23] In an alternate setting, where one state is faced with an existential threat and nuclear weapons are used as a last resort, the Waltz (1981) comments seem highly plausible.

[24] Admittedly, Waltz (1981) may be discussing tactical nuclear weapons. I discuss this in depth below.

parameter from $n = 0$ to some $n > 0$. If this is the case, Remark 3 implies that if we compared how history played out from 1950-present ($n > 0$) to a counterfactual history without the development of nuclear weapons ($n = 0$), then we would observe weakly more conventional conflicts in the counterfactual history. And, if we observed a third counterfactual history—where greater nuclear instability existed among states with nuclear weapons—then we would observe even fewer conventional conflicts.

Two forces drive the nuclear peace. First, as nuclear instability increases, a conventional war becomes a worse option for the defender because the risk of a catastrophic nuclear exchange during the conventional war grows. Second, as nuclear instability increases, the set of possible arming levels that could result in war—in other words, arming levels where C would be willing to challenge and D would be willing to escalate if challenged—is shrinking. Essentially, when D arms in preparation to fight a war, D is undertaking a constrained optimization problem. Specifically, when D wants to fight, D selects a level of arming that optimizes how they do in a conventional war over the set of arming levels that results in C being willing to challenge and D wanting to escalate when challenged. Together, as nuclear instability increases, D's objective function produces categorically worse options—because now conventional war is less stable—and the set over which D optimizes shrinks—because some arming levels that previously would have motivated D or C to fight now, under high nuclear instability, no longer motivate D or C to fight.

To give a sense of what these results look like visually, in Figure 4, I include three plots, each with fixed parameters other than n which increases moving down. *Draft note: I'm working on improving this image.

In the top plot, $n = 0$ (i.e. there is no risk of a nuclear exchange) and there is a large range of values where the game ends with one type or both types of D declaring war. In the second (middle) plot, $n = 0.02$, the parameter space where war occurs shrinks. As n increases, it becomes easier to deter C by arming to level p^C (which is decreasing in n), and it makes war worse; this shifts the Deter-Acquiesce and Deter-Deter regions upwards and grows them while shrinking the the War-Acquiesce and War-War regions (the latter is now only a sliver) In the third plot $n = 0.05$; for the highest values of v_C, both types of D optimally select $p = p_0$ and

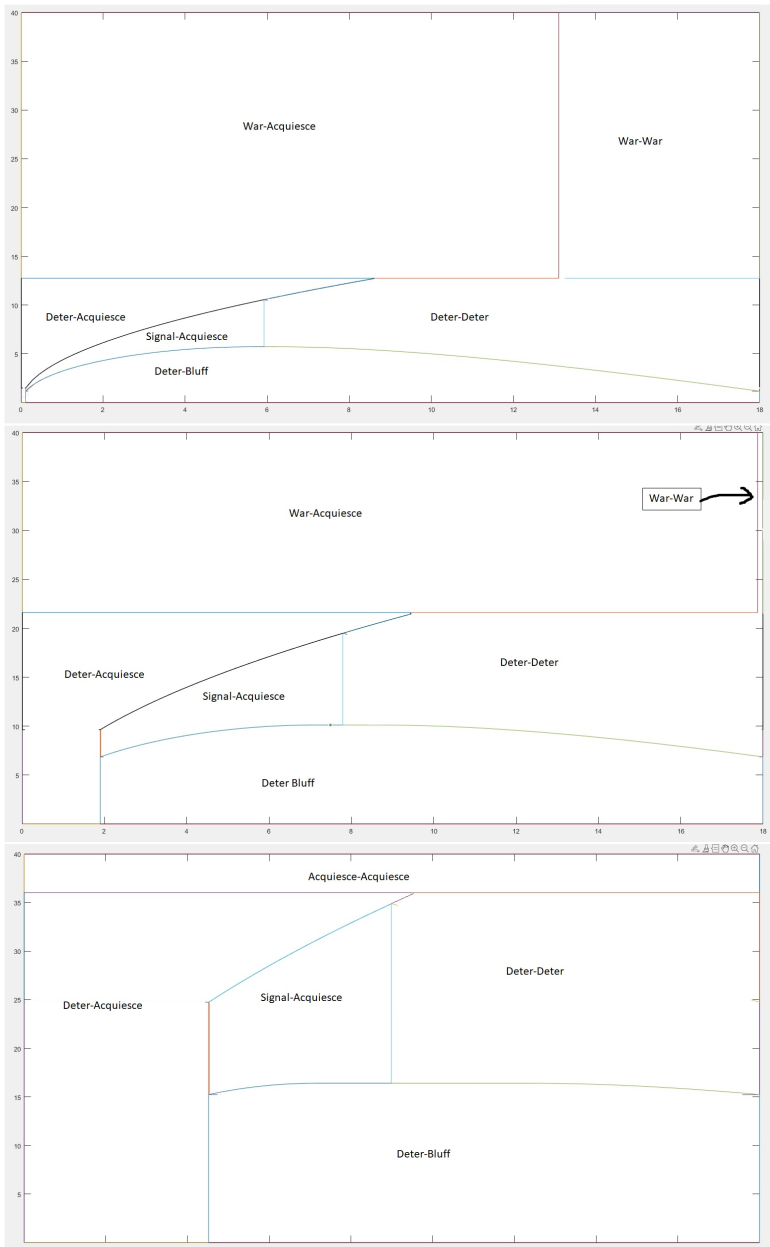

Figure 4: On the x-axis, I vary the values of $\underline{v}_D$, and on the y-axis I vary the values of v_C. The cost function is $k * (p^* - p_0)^2$. The non-illustrated parameter values are $p_0 = 0.001$, $p_1 = 0.8$, $c_D = 2$, $c_C = 1.5$, $N_C = 60$, $N_D = 40$, $\alpha = 0.1$, $k = 15$, $\pi = 0.8$, $\bar{v}_D = 25$. In the top figure, $n = 0$. In the middle figure, $n = 0.02$. In the bottom figure, $n = 0.05$. Apologies the figures are small and crummy, I will fix this in the next iteration

acquiesce. In addition to arming to level p^C becoming cheaper (growing Deter-Acquiesce and Deter-Deter regions), here war becomes more costly for D, and for the greatest values of p^C, both types of D will end up acquiescing to avoid war.

7.4 On Welfare

Remark 4 (Welfare Analysis). *Increasing nuclear instability can lower overall welfare, even in equilibria where war does not occur.*

While increasing nuclear instability (n) does lead to weakly less conventional war (Remark 3), adding nuclear instability is not inherently welfare improving. Increasing nuclear instability can make both actors weakly worse off through two channels. The first channel is straightforward: while adding nuclear instability can reduce the likelihood of war, when it does not rule out war altogether, war can be more costly with greater nuclear escalation risk.[25] This idea is emphasized in work discussing the inherent risks of a world with nuclear instability(Sagan, 1985, 1994). Second, when states do not go to war, increasing nuclear instability can make deterrence more expensive. When D arms to level $p^D(v_D)$, increasing n requires D to arm to a greater level, thus resulting in D attaining a strictly lower utility and C's utility remaining unchanged. This later point is not well examined. Nuclear optimists point to the decrease in great power conflict as a virtue of the nuclear era (De Mesquita and Riker, 1982). This paper suggests that even if there has been a decline in conventional conflict, this decline may have come with added costs of conventional arming, which, by historical standards, are quite high.

7.5 Nuclear Instability, Costs, and Arming Incentives

Remark 5 (Stability-Instability Paradox): *Conditional on D fighting, as n and N_D increase, D will select an arming level that results in a more decisive conflict. Formally, suppose under n and N_D D will select interior arming level $p^* = \hat{p}(v_D)$,*[26] *and that D will go to war.*

(a) If $p^* < \frac{1}{2}$, then $p^*(N_D)$ is decreasing. If $p^* > \frac{1}{2}$, then $p^*(N_D)$ is increasing.

(b) If p^* is small (see Appendix for more details), then $p^*(n)$ is decreasing. If p^* is large, then $p^*(n)$ is increasing.

[25] Naturally, as n increases, D may select a new p^*. However, sometimes escalation risk will still increase.

[26] *Formally* $p \in \left(max\left\{p^D(\bar{v}_D), p_0\right\}, min\left\{p^C, p_1\right\}\right)$

The stability-instability paradox remains a caveat on the concept of a nuclear peace (Snyder, 1965; Jervis, 1984). Empirically, there is some evidence that while the great powers avoided direct, large-scale conventional conflict, they did engage frequently at lower levels (Rauchhaus, 2009; Early and Asal, 2018). That said, the scope conditions for the paradox outside of nuclear-level stability is still an open topic (O'Neill, 2019).

Because nuclear instability within conventional conflict generates new costs for entering into a prolonged conventional conflict, the incentive to avoid prolonged conventional conflict drives strategic behavior. Because greater levels of arming can make the conventional conflict more or less prolonged, D will sometimes have an incentive to invest more and at other times an incentive to invest less. Figure 5 visualizes these effects.

The left graph in Figure 5 illustrates the equilibrium arming level (y-axis) for a range of $\underline{v}_D$ values (x-axis) under a set of parameters where low-type D's always fight. The different dot styles capture changes in D's nuclear cost N_D. The open dots are the arming levels when $N_D = 10$, and the closed dots are when $N_D = 30$. The right graph in figure 5 considers changes in nuclear instability n. The open dots are the arming levels at when $n = 0$, and the closed dots are for $n = 0.015$.

Take the open dots on the left graph of Figure 5, representing the equilibrium arming levels for various $\underline{v}_D$ for $N_D = 10$. As N_D increases (moving from open to closed dots at a given $\underline{v_D}$), equilibrium arming levels below the initial arming level of $p^* = 0.5$ decrease and equilibrium arming levels above the initial arming level of $p^* = 0.5$ increase. For example, for $v_D = 11$ and $N_D = 10$, D will arm to level $p^* = 0.444$. Then, as the nuclear costs increase to $N_D = 30$, D is incentivised to make the conflict more decisive, which D does by selecting a lower arming level $p^* = 422$. For all arming levels $p^* \geq 0.5$, the way to create a more decisive conflict is inverted. At $\underline{v}_D = 20$, under low nuclear instability D will select $p^* = 0.61$, and under high instability ($N_D = 30$) D will select $p^* = 0.619$.

Now consider the open dots on the right graph of Figure 5, representing the equilibrium arming levels for various $\underline{v}_D$. As n increases (moving from open to closed dots), equilibrium arming levels shift in a similar manner to those on the left, but no longer around the same inflection point of $p^* = 0.5$. Why? In short, this is a more complicated relationship, as n factors into a

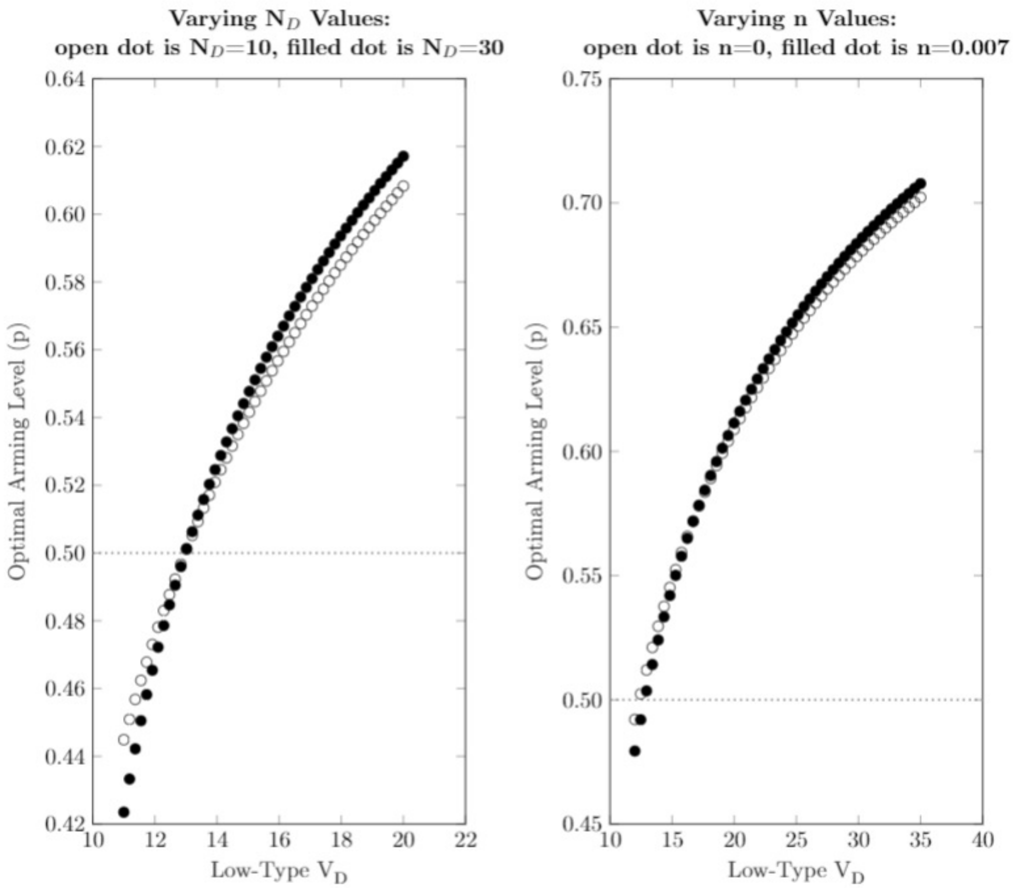

Figure 5: Optimal Arming Levels for D at Different Values of Nuclear Risk

wider range of payoffs; for example, in addition to influencing the cost function, it also changes the likelihood of a conventional versus nuclear conclusion to the conflict. This is not to say this relationship cannot be defined: I express a formula for when increasing n increases or decreases the optimal arming level in the appendix. However, this is a more complicated expression, and depends on a wide range of factors outside of the initial p^*.

7.6 Deterrence Without War

Remark 6: Peaceful signalling of resolve is possible.

A key distinction between equilibrium behavior in this model and the model in Powell (2015) is that here it is possible for the defender to signal their resolve without ever having to go to war.

Within the Signal-Acquiesce region, high-type D's demonstrate their resolve by arming to a level beyond what is needed to make themselves willing to fight, and beyond what is needed to make a conflict sufficiently damaging for C. Formally, here high-type D's select a $p^* > max\{p^D(v_D), p^C\}$. The relevant constraint is that high-type D's must arm to a level where low-resolved D's would be unwilling to mimic high-type D's due to the costs of arming. As a result, in equilibrium, only high-type D's arm to level $\bar{p}$, C knows upon seeing $p = \bar{p}$ that D is a high-type, and C will never challenge. Arming as a costly signal of resolve works all the time, and this equilibrium is peaceful.

As mentioned in Section 4, this is distinct from how costly signalling functions in Powell (2015). In Powell, manipulating nuclear risk is costless unless a war breaks out. And, following the standard signalling logic, unless the signal is costly, low-types are incentivised to mimic high-types and this undermines the informative value of the signal. As a result, in Powell, the defender can only signal their resolve by actually fighting sometimes because only through conventional conflict are the signal's costs realized.[27] In this regard, arming as a costly signal does effecitvely separate low-types from high-types, but the equilibrium is not always peaceful.

[27] In the equilibrium in Powell, the resolved defender must select a level of nuclear risk that makes the challenger indifferent between challenging and not challenging, and the resolved defender must fight with positive probability.

These distinctions have real-world implications. The model here suggests that resolved defenders can deter challengers and entirely prevent conflict through the signal of a costly robust conventional force posture. In contrast, Powell (2015) suggests that manipulating nuclear risk cannot function as a fully effective deterrent against challengers, and that conflict must sometimes break out. There are two ways to interpret these results. First, the model here presents a more optimistic perspective on nuclear deterrence than Powell (2015). In Powell, war is an inevitable part of signalling resolve. In contrast, I find it is possible to signal resolve and deter an opponent without ever having to resort to conflict. Second, from a practical perspective, if a defender wanted to signal resolve and avoid conflict, manipulating nuclear risk (ala Powell) is not as effective as committing troops.

7.7 Additional Results

7.7.1 Deterrence Failure and Nuclear Instability

I classify a "deterrence failure" as any equilibrium where C challenges D. As nuclear instability increases, deterrence failures could become more or less common. Two competing effects drive this. First, sometimes D deters C by arming to level $p = p^D(v_D)$. As n increases, $p^D(v_D)$ also increases, possibly to the point where D is unwilling to undertake the (costly) arming needed to deter C from challenging. If D is no longer willing to deter C from challenging, then the increase in n produces a deterrence failure. Second, sometimes D deters C by arming to level $p = p^C$. As n increases, p^C decreases. In some cases, D may have been unwilling to arm to level p^C and deter C from challenging under under a low level of nuclear stability; however, under greater levels of nuclear instability, which leads to a lower p^C, D would be willing to arm to a level that prevents a deterrence failure. Ultimately, whether D experiences more or less deterrence failures following increases in nuclear instability depends on the underlying conditions of the case.

These findings pose a useful counterpoint to Mearsheimer (1990), which states that "[d]eterrence is most likely to hold when the costs and risks of going to war are unambiguously stark."[28] Using

[28]The model confirms that deterrence is more likely to hold when the costs of a nuclear war for the challenger (N_C) are high, but deterrence is more likely to fail when the costs of a nuclear war for the defender (N_D) are high.

the definition of deterrence failures here, the model suggests this is not necessarily the case. When there is a high nuclear risk, it could undermine D's deterrent threat and encourage C to challenge.

7.7.2 Caveats to the Nuclear Peace: Effects Beyond Nuclear Instability

Understanding the true effect of the nuclear revolution on conventional conflict is potentially more complex than what is presented above. This model finds that increasing the nuclear instability parameter n always results in less conventional conflict (Remark 3). A natural interpretation of the nuclear revolution is that the world changed from $n = 0$ to $n > 0$ and, as a result, engaged in less conflict.

However, the nuclear revolution has also shaped conventional forces in ways that influence the likelihood of conventional conflict. Today, several states deploy nuclear powered submarines and aircraft carriers. Additionally, it is possible that one day tactical nuclear weapons are deployed on the battlefield as part of a "conventional" conflict. Together, it could be claimed that technologies like nuclear submarines, because they are more efficient and capable then their non-fission powered counterparts, could lower the costs of conducting a conventional war (i.e. it reduces c_D and c_C). Lowering c_D and c_C have the effect of increasing the parameter space under which conventional conflict occurs. The rationale is essentially the opposite of what is shown in Remark 3: as D's (or C's) costs from war decrease, conditional on fighting, D's objective function increases at all points, and the set over which D optimizes expands.

To the extent that the nuclear revolution both increased nuclear instability and lowered the costs of conventional war, the true effect of the nuclear revolution requires disentangling competing effects. Consider, for example, the possibility that the challenger develops and would deploy tactical nuclear weapons, and the deployment of these technologies increases nuclear instability. When this occurs, evaluating the nuclear peace hypothesis becomes more complex. On one hand, increases to the nuclear instability parameter would still, at a fixed arming level, make a conventional war more costly. On the other hand, if the challenger could conduct conflict more easily, this expands the set of arming levels that would eventually result in a conventional crisis. Without additional structure, I am unable to claim that a nuclear peace result would hold following the development of nuclear weapons.

8 Extensions

8.1 Burning Bridges

Schelling (1966, pp. 43–44) describes the strategic logic of burning bridges. To summarize, states want to "burn their bridges" to make the options that are not fighting worse, which in turn removes doubt in the challenger's mind that their advance will not be met with a fight. I modify the model above to allow for an analysis of the burning bridges logic. In the model above, if D selects some conventional arming level p^* then acquiesces, then the costs of arming are treated as sunk. In an extension developed in the Appendix, I examine a version of the model where D's costs are not fully sunk; in other words, in the new model, when C challenges then D acquiesces, D can recover some of these costs (Draft note: I have not done this). This is analogous to D being able to demobilize and retreat while acquiescing; in other words, when D can recover some arming costs, D has not burned their bridges.

In the new model where D can recover some of their arming if they acquiesce, D does worse or the same across all equilibria spaces. Why? The ability for D to recover their arming upon acquiescing makes acquiescing more appealing than fighting. This means that in this new model, the cutpoint level of arming that D needs in order to be willing to fight (p^D) increases. This means whenever D "deters" by arming to level $p^D(v_D)$ (with $v_D \in \{\underline{v}_D, \bar{v}_D\}$), D must spend more. Additionally, this modification shrinks the set over which D optimizes when D fights.

Interestingly, this modification does not change the equilibrium with signalling. In this equilibrium, high-type D's must arm to a level where low-type D's would not want to arm to conditional on them getting the asset. Essentially, burning bridges (or failure to burn bridges) does not matter here. Where the burning bridges option matters is when D arms to a level $p^* > p_0$, is challenged, and then retreats. However, this never occurs in the signalling space due to a combination of (a) the intuitive criterion and (b) that a full separation of types is feasible.

8.2 Endogenously Arming Challenger

Pending

8.3 Covert Operations

Pending.

9 Conclusion

Every day, every human on earth lives with the background risk of a catastrophic nuclear exchange (Sagan, 1985). While I do not wish to discount this prospect, this paper suggests that neorealist logic may be correct, and that this latent instability comes with the benefit of reducing the likelihood of a conventional war. To this end, I have demonstrated that the observed "long peace" could be a "nuclear peace," where the nuclear great powers are less willing to engage in large conventional wars and more willing to engage in small, regional contests with small risk of nuclear escalation.

Of course, as this paper has demonstrated, this latent nuclear risk does not come without costs. While much attention has been paid to the underlying risks that nuclear weapons hold (Sagan and Waltz, 1995), this paper demonstrates that nuclear weapons can reduce welfare through other channels as well. In a nuclear world, deterrence may become more difficult and costly: states might find themselves investing more in conventional armaments to make their threat to fight a conventional war that could end in disaster credible. And, even the prospect of a "nuclear peace" is subject to caveats; to the extent that nuclear technologies make conventional conflict cheaper (through, for example, the development of nuclear-powered naval vessels), the connection between nuclear technologies and peace is more open to debate.

There are still many research avenues on this topic to consider. This paper treats several variables as exogenous when, in reality, they are strategic choices. For example, while I believe it is difficult to credibly manipulate nuclear instability within a crisis, bigger picture, this paper does not consider how states optimally design nuclear instability based on what crises they expect with deterrence in mind. Additionally, this paper treats the challenger's arming levels as fixed when these are plausibly endogenous. Finally, this paper adopted a specific functional form for wartime payoffs. Future research could either (a) generalize this, or (b) use the framework here to work within a continuous time conflict framework.

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